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by Joseph Ajello

Prepared under Grant No. NsG-48-60 *by*
RENSSELAER POLYTECHNIC INSTITUTE
Troy, New York
for

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ABSTRACT

The purpose of this experiment was to measure the probability of collision for momentum transfer of slow electrons in a weakly ionized nitrogen plasma afterglow. The probability of collision was calculated in the late afterglow on the basis of a hard sphere model interaction between electrons and neutrals. The value obtained over the energy range .040 ev to .078 ev is 18.1 ± 1.6 (cm Torr)$^{-1}$. The time taken by the electron gas to relax from the high temperature of the active discharge to room temperature and to a Maxwellian distribution is found to be much longer than that predicted by a consideration of elastic collisions between electrons and neutrals. As expected, the rate of relaxation to room temperature and to a Maxwellian distribution is a sharp function of pressure. The reason for these effects is ascribed to the disturbance of the afterglow by interactions involving metastable nitrogen particles.

AUTHOR

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PART I

INTRODUCTION

The purpose of this investigation was to determine experimentally the probability of collision for momentum transfer of slow electrons in a nitrogen plasma afterglow. It is hoped that these results will resolve some of the discrepancy that exists in the measured values of the probability of collision for electrons in nitrogen in the low energy range.[18] The experimental technique used was novel in that the apparatus has the ability to monitor electron temperature as a function of time. All previous measurements were interpreted to have been made after the time necessary for the electron energy to decrease to room temperature. This time has generally been estimated on the basis of elastic collisions between the electrons and neutral particles. Such a time is almost certainly too low in nitrogen, owing to the presence of an energy source which acts to increase the energy of the electrons. This source is the excitation energy of long-lived metastable molecules. Most probably therefore, some of the results reported by other workers as taking place at room temperature actually involved higher energy electrons. The microwave radiometer which measures the electron temperature as a function of time is indispensable for a thorough analysis of an afterglow.

Our objective also requires that we measure the plasma conductivity as late into the afterglow as possible. The apparatus used for the determination of the plasma conductivity is a microwave interferometer or bridge. In this device the microwave signal is split into two parts. One part propagates through the arm of the bridge which contains the plasma and enters the mixing section. The other part enters the mixing section from the opposite direction. The consequent interference pattern is detected and displayed on an oscilloscope. The phase shift and attenuation introduced by the plasma can be measured in this way and the plasma conductivity can be calculated. The collision frequency is obtained from the measured conductivity and is related to the probability of collision through the expression $\nu_m(v) = v\ p_o\ P_c(v)$

where ν_m is the collision frequency for momentum transfer between the electrons and neutral particles

v is the electron velocity

p_o is the pressure normalized to $0^{o}C$

P_c is the probability of collision

The microwave signal is of such a low level that it does not disturb the plasma by heating the electrons. Meaningful measurements were made from 50 microseconds to 200 microseconds into the afterglow. After this time the attenuation due to the plasma was too weak to be recorded although phase shift-data can be taken for a longer period.

PART II

THEORY AND INSTRUMENTATION

A. Microwave Conductivity of a Plasma

The conductivity of a free electron gas in a vacuum is ninety degrees out of phase with the electric field. This fact implies that the alternating electric field does not impart energy on the average to the electron. In a weakly ionized plasma the ordered motion of the electrons in the electric field is disturbed by collisions with neutral particles. The plasma conductivity thereby acquires a component which is in phase with the electric field, indicating an energy transfer from the electric field to the random motion of the electrons.

Two approaches may be made to the derivation of a formula for the conductivity. The more rigorous approach is based upon the use of the Boltzmann equation of kinetic theory. A simpler approach assumes that all the electrons behave in the same way as an "average" electron and procedes on the basis of the Langevin equation. We discuss this latter approach first.

1. The Microwave Conductivity from the Average Electron Theory

The conductivity for the plasma is obtained by setting the total rate of change of momentum of the average electron in the electric field direction equal to the rate of change of momentum due to the field, plus the rate of change of momentum in the field direction produced by collisions[8].

$$-e\,E_o \exp j\omega t = \frac{d(m\langle v\rangle)}{dt} + \nu_m\, m\langle v\rangle \qquad (1)$$

where

E_o is the maximum amplitude of the electric field

ω is the angular frequency of the electric field

$\langle v \rangle$ is the average electron velocity.

It follows that

$$\langle v \rangle = -e E_o \exp j\omega t / m(\nu_m + j\omega)$$

The current density J is given by

$$J = -ne\langle v \rangle = \frac{ne^2 E_o \exp j\omega t}{m(\nu_m + j\omega)} \tag{2}$$

where n is the number density of the electrons. The conductivity σ is a complex quantity which can be split into its real and imaginary parts and written as

$$\sigma = \frac{J}{E} = \sigma_r + j\sigma_i \tag{3}$$

The expressions for σ_r and σ_i can be obtained from (2) and (3) and are

$$\sigma_r = \frac{ne^2}{m\omega} \frac{\nu_m/\omega}{(\nu_m/\omega)^2 + 1}$$

and

$$\sigma_i = -\frac{ne^2}{m\omega} \frac{1}{(\nu_m/\omega)^2 + 1}$$

The ratio σ_r/σ_i is therefore

$$\frac{\sigma_r}{\sigma_i} = -\frac{\nu_m}{\omega} \tag{4}$$

This relation is independent of electron density.

2. The Microwave Conductivity from the Boltzmann Equation

We will give here an expanded version of the theory due to Phelps et al.[17]

The Boltzmann equation for the electrons is given by[4]

$$\frac{\partial f}{\partial t} + \bar{a} \cdot \bar{\nabla}_v f + \bar{v} \cdot \bar{\nabla}_r f = \left(\frac{\partial f}{\partial t}\right)_{coll} \qquad (5)$$

where f is the electron velocity distribution function and

$\bar{a}$ is the acceleration of the electrons.

The plasma density is assumed to be spatially uniform so that f is independent of the position coordinate and the third term of (5) can be equated to zero.

We now assume a form for f given by

$$f = f_o + f_1 \qquad (6)$$

where f_o is the Maxwellian distribution function and

f_1 is the perturbation term due to the applied fields.

If (6) is substituted into (5) we obtain two terms on the right hand side. $(\partial f_o/\partial t)_{coll}$ is equal to zero because the Maxwellian distribution is the equilibrium distribution. $(\partial f_1/\partial t)_{coll}$ can be written as $-f_1 \nu_m$, that is, the rate of destruction of the preferred motion of the electrons. This assumes that the collision frequency for momentum transfer ν_m expresses the rate at which electrons lose their preferred motion. ν_m is usually a function of the electron velocity.

If an electromagnetic wave at a microwave frequency is propagating through the plasma an alternating electric field is present which is given by

$$\bar{E} = \bar{E}_o e^{j\omega t} \qquad (7)$$

so that the acceleration of the electrons $\bar{a}$ is given by

$$\bar{a} = -\frac{e}{m} E_o e^{j\omega t} \tag{8}$$

Inserting (6) in (5) under these conditions we obtain

$$\frac{\partial f_1}{\partial t} - \frac{e}{m} \bar{E}_o e^{j\omega t} \cdot \bar{\nabla}_v f_o = -f_1 \nu_m(v) \tag{9}$$

Now the field induced part of f which is f_1 will have a time variation given by $e^{j\omega t}$ so that

$$\frac{\partial f_1}{\partial t} = j \omega f_1 \tag{10}$$

If we substitute (10) in (9) and solve for f_1 we obtain

$$f_1 = \frac{e}{m} \frac{1}{(\nu_m + j\omega)} \bar{E}_o e^{j\omega t} \cdot \bar{\nabla}_v f_o \tag{11}$$

The current density J in a plasma is

$$J = -ne \langle v \rangle = -e \int_{-\infty}^{\infty} f \, v \, dv \tag{12}$$

If equation (6) is substituted into (12) we obtain terms in f_o and f_1. Now f_o is the Maxwellian distribution function which is an even function of v so that $\int_{-\infty}^{\infty} f_o \, v \, dv$ is an odd function of v and is therefore equal to zero.

The current density thus becomes

$$J = -e \int_{-\infty}^{\infty} v \, f_1 \, dv \tag{13}$$

Substituting (11) into (13) gives

$$J = -\frac{e^2}{m} \bar{E}_o e^{j\omega t} \int_{-\infty}^{\infty} \frac{\nu}{\nu + j\omega} \bar{1}_F \cdot \bar{\nabla}_v f_o \, dv \tag{14}$$

where ν has been written for ν_m.

If we assume that $\bar{1}_F$, the unit vector in the field direction, is directed along the x-axis, σ is then the conductivity in the x direction. It is given by the relation

$$\sigma = \frac{J}{E} = - \frac{ne\langle v_x \rangle}{E} = - \frac{e^2}{m} \int_{-\infty}^{\infty} \frac{v_x}{\nu + j\omega} \frac{\partial}{\partial v_x} f_o \, dv \qquad (15)$$

For a nearly isotropic distribution function the following two relations are easily deduced

$$\frac{\partial f_o}{\partial v_x} = \frac{v_x}{v} \frac{\partial f_o}{\partial v} \qquad (16)$$

$$v_x^2 = \frac{v^2}{3} \qquad (17)$$

Substituting (16) and (17) into (15) we obtain

$$\sigma = - \frac{4\pi}{3} \frac{e^2}{m} \int_o^{\infty} \frac{1}{\nu + j\omega} v^3 \frac{\partial}{\partial v} f_o \, dv \qquad (18)$$

The ratio of the real to the imaginary part of the conductivity from equation (18) gives

$$\frac{\sigma_r}{\sigma_i} = \frac{\displaystyle\int_o^{\infty} \frac{\nu v^3 \frac{\partial f_o}{\partial v}}{1 + (\frac{\nu}{\omega})^2} dv}{\displaystyle\int_o^{\infty} \frac{\omega v^3 \frac{\partial f_o}{\partial v}}{1 + (\frac{\nu}{\omega})^2} dv} \qquad (19)$$

This relation is independent of electron density.

As the distribution function f_o is Maxwellian, then

$$f_o(v) = n \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-1/2 \frac{m v^2}{kT}}$$

which can be written in the form

$$f_o(v) = A\, e^{-\frac{u}{w}} \tag{20}$$

where

u is $(1/2)\, m\, v^2$

w is kT

A is $n\left(\frac{m}{2\pi\, kT}\right)^{3/2}$

T is the electron temperature.

Substituting (20) into (19) gives

$$\frac{\sigma_r}{\sigma_i} = \frac{\displaystyle\int_o^\infty \frac{\nu}{\omega}\, \frac{u^{1.5} \exp\left(-\frac{u}{w}\right)}{\left(\frac{\nu}{\omega}\right)^2 + 1}\, du}{\displaystyle\int_o^\infty \frac{u^{1.5} \exp\left(-\frac{u}{w}\right)}{\left(\frac{\nu}{\omega}\right)^2 + 1}\, du} \tag{21}$$

When $\omega^2 \gg \nu^2$ equation (21) becomes

$$\frac{\sigma_r}{\sigma_i} = \frac{-\displaystyle\int_o^\infty \frac{\nu}{\omega}\, u^{1.5} \exp\left(-\frac{u}{w}\right) du}{\displaystyle\int_o^\infty u^{1.5} \exp\left(-\frac{u}{w}\right) du} \tag{22}$$

The denominator of (22) can be evaluated as follows

$$\int_o^\infty u^{1.5} \exp\left(-\frac{u}{w}\right) du = w^{2.5}\, \frac{3\pi^{1/2}}{4} \tag{23}$$

Put (23) into (22) to obtain

$$-\frac{\sigma_r}{\sigma_i} = \frac{4}{3\pi^{1/2}\, w^{2.5}} \int_o^\infty \frac{\nu}{\omega}\, u^{1.5} \exp\left(-\frac{u}{w}\right) du \tag{24}$$

where we bear in mind that ν is a function of u.

It is known from the theory of linear integral equations, that this integral can be solved approximately by a finite polynomial of the first kind.[10] We therefore assume that σ_r/σ_i can be expanded in the form

$$\frac{\sigma_r}{\sigma_i} = - \sum_{j}^{\ell} a_j (w^{1/2})^j \quad p_o \tag{25}$$

where j and ℓ are integers. Equation (24) can then be written

$$-w^{2.5} \frac{\sigma_r}{\sigma_i} = \int_o^\infty \frac{4}{3\pi^{1/2}} \frac{\nu}{\omega} u^{1.5} \exp - \frac{u}{w} \, du \tag{26}$$

This equation is in the form of a Laplace transform, if the following substitutions are made.[17]

$$w = \frac{1}{p}$$

$$g(p) = - \frac{1}{p^{2.5}} \frac{\sigma_r}{\sigma_i} \tag{27}$$

$$f(u) = \frac{4}{3\pi^{1/2}} \frac{\nu}{\omega} u^{1.5} \tag{28}$$

Equation (26) may now be written

$$g(p) = \int_o^\infty e^{-pu} \, du \tag{29}$$

Taking the inverse Laplace transform of (29) we obtain

$$f(u) = L^{-1} g(p) \tag{30}$$

where L^{-1} is an inverse Laplace transform operator. Substitute relations

(25) and (27) into (30) and obtain

$$f(u) = L^{-1} \sum_{j}^{\ell} a_j \left[\left(\frac{1}{p}\right)^{.5} \right]^j \frac{1}{p^{2.5}} \; p_o$$

which can be written as

$$f(u) = \sum_{j}^{\ell} a_j \; p_o \; L^{-1} \frac{1}{p^{2.5+.5j}}$$

If $2.5 + .5j$ is an integer then j is an odd number and

$$f(u) = \sum_{j}^{\ell} a_j \frac{u^{(1.5 + .5j)}}{(1.5 + .5j)!} \; p_o \tag{31}$$

Equation (31) can be written as follows

$$\frac{4}{3\pi^{1/2}} \; \frac{\nu}{\omega} \; u^{1.5} = \sum_{j}^{\ell} \frac{a_j \; u^{(1.5 + .5j)}}{\left(\frac{3+j}{2}\right)!}$$

Solving for $\nu(u)$ we obtain

$$\nu \,(u) = p_o \; \frac{3\pi^{1/2}}{4} \; \omega \sum_{j}^{\ell} \frac{u^{.5j}}{\left(\frac{3+j}{2}\right)!}$$

In particular let

$$\frac{\sigma_r}{\sigma_i} \;(w) = -p_o \left[a_1 < u >^{1/2} + a_2 < u >^{3/2} + a_3 < u >^{5/2} \right] \tag{32}$$

then

$$\nu = \frac{3}{4} \pi^{1/2} \; \omega \left[\frac{a_1}{2} \; u^{1/2} + \frac{a_2}{6} \; u^{3/2} + \frac{a_3}{24} \; u^{5/2} \right] \tag{33}$$

Since $u = \frac{1}{2} m v^2$ then

$$v = (\frac{u}{2m})^{1/2}$$

If u is in electron volts then

$$v = \frac{u^{1/2}}{1.69 \times 10^{-8}} \quad \text{in units of } (\text{eV})^{1/2}$$

and because $\nu = v \, p_o P_c$, (34)

$$P_c = \frac{1.69 \times 10^{-8}}{u^{1/2} p_o} \nu \quad (\text{cm-Torr})^{-1}$$

Substitute equation (33) into (34) to obtain

$$P_c = \frac{1.69 \times 10^{-8}}{u^{1/2}} \frac{3}{4} \sqrt{\pi} \quad \omega \left[\frac{a_1}{2} u^{1/2} + \frac{a_2}{6} u^{3/2} + \frac{a_3}{24} u^{5/2} \right]$$

and

$$P_c = 1.69 \times 10^{-8} \frac{3}{4} \sqrt{\pi} \quad \omega \left[\frac{a_1}{2} + \frac{a_2}{6} u + \frac{a_3}{24} u^2 \right] \tag{35}$$

Therefore if a $\frac{\sigma_r}{\sigma_i}$ versus w curve is plotted and three points whose projections on the w axis are equally spaced are chosen, a_1, a_2, and a_3 can be determined. P_c is then known over this energy range. Note once again that the assumption of a Maxwellian distribution for the electrons has been made in this derivation.

3. The Conductivity of a Plasma for Two Special Cases

Two special cases are interesting becuase they are soluble and represent physically realizable situations. The case of constant mean free path represents the situation for a billiard ball or hard sphere interaction. This is an excellent approximation for slow electrons in a plasma afterglow.[15] The case of constant collision frequency agrees with the results of the average electron theory. It represents molecules repelling each other with a force varying inversely as the fifth power of the separation distance. Molecules obeying this law are called Maxwellian molecules.

a. Constant Mean Free Path

Margenau[13] has also derived an expression for the a.c. conductivity of a plasma starting from the Boltzmann equation. He obtains an expression for σ similar to (18) above. Making the constant mean free path assumption, the collision frequency ν can be expressed as a function of velocity by

$$\nu = \frac{v}{L_{const}} \quad \text{or simply} \quad \nu = \frac{1}{L}\, v \tag{36}$$

where L is the mean free path,
so that the integration over velocity space can be carried out (refer to equation (18)). If, in addition, the symmetric part of the velocity distribution function is taken to be the Maxwellian distribution then Margenau[13] obtains

$$\sigma = \frac{4}{3}\, \frac{e^2\, L\, n}{(2\pi\, mkT)^{1/2}} \left[K_2(x_1) - j\, x_1^{1/2}\, K_{3/2}(x_1) \right] \tag{37}$$

The functions $K_2(x_1)$ and $K_{3/2}(x_1)$ can be expressed in terms of the exponential integral $Ei(-x_1)$ and the error function $\mathrm{erf}\left[(x_1)^{1/2}\right]$ as follows

$$K_2(x_1) = 1 - x_1 - x_1^2 \exp x_1 \; Ei(-x_1)$$

$$K_{3/2}(x_1) = (\tfrac{1}{2} - x_1)\, \pi^{1/2} + \pi^{1/2}\, x_1^{3/2} \exp(x_1) \left[1 - \mathrm{erf}(x_1)^{1/2}\right]$$

The two following asymptotic forms for the real and imaginary parts of the conductivity are given by Oskam.[16]

For the case when $\omega^2 >> <\nu>^2$

$$\sigma_r(t) = \frac{4}{3} \frac{e^2 n(t)}{m\,\omega^2} <\nu> \left[1 - \frac{3\pi}{4} \left(\frac{<\nu>}{\omega}\right)^2 \dots\right] \qquad (38)$$

$$\sigma_i(t) = - \frac{e^2 n(t)}{m\,\omega} \left[1 - \frac{5\pi}{8} \left(\frac{<\nu>}{\omega}\right)^2 \dots\right] \qquad (39)$$

where $<\nu> = \frac{<v>}{L} = \frac{1}{L}\left(\frac{8kT}{\pi m}\right)^{1/2}$ for a Maxwellian distribution.

And for the case when $<\nu>^2 >> \omega^2$

$$\sigma_r = \frac{8}{3\pi} \frac{e^2 n(t)}{m} \frac{1}{<\nu>} \left[1 - \frac{4}{\pi} \left(\frac{\omega}{<\nu>}\right)^2 \dots\right] \qquad (40)$$

$$\sigma_i = - \frac{8}{3\pi} \frac{e^2 n(t)}{m} \frac{\omega}{<\nu>^2} \left[1 - \frac{8}{\pi} \left(\frac{\omega}{<\nu>}\right)^2 \dots\right] \qquad (41)$$

The ratio of the real to the imaginary components of the conductivity can be asymptotically obtained to first order in $<\nu>/\omega$ for these two cases as

$$\frac{\sigma_r}{\sigma_i} \sim -\frac{4}{3} \frac{<\nu>}{\omega}, \qquad \omega^2 >> <\nu>^2 \qquad (42)$$

$$\frac{\sigma_r}{\sigma_i} \sim -\frac{<\nu>}{\omega}, \qquad <\nu>^2 >> \omega^2 \qquad (43)$$

where $\sim$ is an asymptotic symbol.

The case of interest to us in the present work fulfills the condition that $\omega^2 >> <\nu>^2$.

b. Constant Collision Frequency

The conductivity for this case is easily obtained by integrating equation (18) by assuming a Maxwellian velocity distribution.

$$\sigma = \frac{e^2 n(r,t)}{m\,\omega} \quad \frac{1}{1+(\nu/\omega)^2} \left[\frac{\nu}{\omega} - j \right] \tag{44}$$

The ratio of the real to the imaginary parts of the conductivity yield

$$\frac{\sigma_r}{\sigma_i} \sim -\frac{\nu}{\omega}\,, \qquad \omega^2 >> \nu^2 \tag{45}$$

This is the same result as obtained for the average electron conductivity.

B. Cross-Sections and Probability of Collision

The average number of collisions per unit time undergone by an electron of speed v, with molecules of all other speeds, is called the collision frequency of an electron of speed v. It is expressed mathematically by the relation

$$\nu_c = \frac{v}{L(v)} \tag{46}$$

where $L(v)$ is the mean free path of electrons of speed v and ν_c is the collision frequency of electrons with molecules

To a good approximation slow electrons in a weakly ionized gas interact with the molecules in a billiard ball type elastic collision. Electrons having an energy above the first excitation level of the gas (vibrational or rotational) will lose their excess energy almost at once to the neutral atoms by inelastic collisions. Thus in an afterglow the number of inelastic collisions is negligible with respect to elastic collisions with molecules in their ground state.

Collision frequencies measured by microwave techniques are averaged over the velocity distribution function of the colliding particles, so that

$$< \nu_c > = \int \nu \, f \, dc \tag{47}$$

Now we can write

$$\nu_c = N \, v \, Q \tag{48}$$

where Q is the molecular collision cross-section and

N is the number density of the molecules.

It is therefore possible to write $< \nu_c >$ as

$$< \nu_c > = N < v > Q \tag{49}$$

The expression $< \nu_c > = N < v > Q$ is only correct if Q can be considered constant for all values of the velocity. Another useful approximation is to consider ν_c a constant over the velocity range of interest. Then

$$< \nu_c > = \nu_c \tag{50}$$

This implies $Q = \frac{a}{v}$, where a is a proportionality constant. We then obtain

$$\nu_c = N \, v \, Q = N \, v \, \frac{a}{v} = N \, a$$

or

$$a = \frac{\nu_c}{N}$$

For an elastic collision between an electron and a neutral molecule, the speed of the molecule is essentially unchanged. The change in momentum of the electron along its initial path is

$$\Delta \text{ Momentum} = \frac{m \, M}{m+M} (1-\cos \theta)$$

and the fractional change in energy is

$$\frac{\Delta \text{ Energy}}{\text{Energy}} = 2 \frac{m M}{(m+M)^2} (1-\cos \theta) = \frac{\Delta u}{u}$$

where θ is the angle between the incident and scattered directions of the electron.

Now $$\frac{\Delta u}{u} = \frac{2m}{M} (1-\cos \theta) \qquad \text{if } M >> m .$$

If $p(\theta) \sin \theta \, d\theta \, d\varphi$ is the probability that, due to a collision the electron will be scattered into the solid angle $d\Omega$ about θ, the mean fractional energy loss per collision will be

$$\overline{\left(\frac{\Delta u}{u}\right)} = \frac{2m}{M} \int_0^{\pi} \int_0^{2\pi} (1-\cos \theta) \, p(\theta) \sin \theta \, d\theta \, d\varphi \qquad (51)$$

$p(\theta) \, Q \sin \theta \, d\theta \, d\varphi$ is called the differential cross-section for elastic scattering into the solid angle $d\Omega$.[14] It is written

$$I(v,\theta) \sin \theta \, d\theta \, d\varphi$$

or

$$p(\theta) = \frac{I(\theta)}{Q} \qquad (52)$$

and

$$Q = \int_0^{\pi} \int_0^{2\pi} I(v,\theta) \sin \theta \, d\theta \, d\varphi$$

The mean fractional energy loss per collision becomes $\frac{2m}{M} \frac{Q_m}{Q}$

where

$$Q_m(v) = \int_0^{\pi} \int_0^{2\pi} I(v,\theta) \, (1-\cos \theta) \sin \theta \, d\theta \, d\varphi \qquad (53)$$

Q_m differs from Q only if there is a pronounced concentration of scattering in either the forward or the backward directions.

If the model of collision is that of hard spheres $Q_m = Q$ and

$\nu_m = \nu_c$. This interaction denotes $I(\theta)$ independent of θ. For slow electrons this is a good approximation in the absence of Ramsauer effects. Now let us write

$$\nu_m = N\, Q_m\, v$$

where $L_m(v)$ is the mean free path for momentum transfer and

L_m is defined by $L_m \equiv \frac{1}{N\, Q_m}$

Then

$$\nu_m(v) = \frac{v}{L_m(v)} = N\, v\, Q_m(v) \tag{54}$$

Equation (54) agrees with the previous definition, $\nu_m = v\, p_o P_c$, if

$$p_o\, P_c(v) = N\, Q_m(v) \tag{55}$$

P_c, the momentum transfer collision probability, takes into account the fact that the effectiveness of collisions in resisting current flow increases as the scattering angle increases.

C. Recombination

In a plasma the fundamental loss processes are diffusion, recombination, and attachment. This can be expressed mathematically

$$\frac{\partial n}{\partial t} = D\, \nabla^2 n - \nu_a n - \alpha_e\, n\, n_+ \tag{56}$$

where n is the electron density

n_+ is the ion density

D is the ambipolar diffusion coefficient for electrons and ions

α is the recombination coefficient

ν_a is the attachment coefficient.

Attachment takes place only in strongly electronegative gases such as oxygen and can be assumed to be zero in our case. Also diffusion losses are negligible with respect to recombination for approximately the first 160 microseconds under the experimental conditions encountered here. This fact has been experimentally demonstrated by Stotz.[18]

$$\therefore \quad \frac{\partial n}{\partial t} = -\alpha \, n \, n_+ = -\alpha \, n^2 \tag{57}$$

Now the electron and ion densities are equal so that (57) can be solved for the electron density as follows

$$\frac{1}{n} = \frac{1}{n_o} + \alpha \, t \tag{58}$$

where n_o is the electron concentration at $t = 0$.

From equation (58) the following result is obtained

$$\frac{dn/dt}{n} = -\alpha \, n \tag{59}$$

The relative change in the electron density is proportional to the density. Therefore the relative change in density is greatest in the more dense regions of the plasma. This effect tends to make the electron density uniform. The prevalent recombination process in the nitrogen afterglow seems to be[18]

$$N_2 + N_2^+ \longrightarrow N_4^+$$

$$N_4^+ + e \longrightarrow N_2 + N_2^*$$

The asterisk denotes an excited atom. This is a two-step dissociative process.

D. Radiation Temperature Measurements

It can be shown that the ratio of the emission coefficient P_ω to the absorption coefficients A_ω equals the blackbody intensity $B(\omega,T)$[11], assuming a Maxwellian distribution of electron velocities. That is,

$$\frac{P_\omega}{A_\omega} = B(\omega,T) \tag{60}$$

Since P_ω and A_ω are properties of the radiating electrons, equation (60) should be true whenever the electrons have a Maxwellian distribution. No other criteria, such as exact thermal equilibrium, need be imposed.

The radiation at microwave frequencies from plasmas is caused by individual or collective motions of free electrons. The thermal radiation originates from the free electrons as they move about in the plasma, colliding with ions and neutral particles. As a result of an interaction a photon of frequency ω is either emitted or absorbed. The frequency of the photon is given by the Bohr frequency condition.

$$\hbar\,\omega = \frac{1}{2} m \; v''^2 - \frac{1}{2} m\, v'^2 \tag{61}$$

v'' is the final electron velocity in center-of-mass coordinate system

v' is the initial electron velocity in the center-of-mass coordinate system

$B(\omega T)$, the intensity of the radiation field, is given everywhere within the container by the Planck radiation formula (for one polarization).

$$B(\omega,T) = \frac{n\,\omega^2}{8\pi^3 c^2} \left[\exp\frac{\hbar\,\omega}{k\,T} - 1\right]^{-1}$$

In the Rayleigh Jeans limit for low frequency $kT >> h\,\omega$ the exponential can be expanded in a power series and simplified as follows

$$S_\omega = \text{source function} = B(\omega,T) = \frac{\omega^2}{8\pi^3 c^2} \; k\,T \tag{62}$$

Unless the distribution function is Maxwellian T is a fictitious temperature and is merely a convenient way of describing the field.[5]

In this experiment the plasma radiation temperature is related to the average energy by the equation

$$\text{Average kinetic energy of the electrons } \langle u \rangle = \frac{3}{2} kT$$

The basic method for measuring electron temperature is a microwave pyrometer scheme.[6,19] The advantage of this attack is the determination of electron temperature without knowledge of the absorptivity of the plasma. Kirchoff's radiation law may be stated

$$P_\omega = A_\omega B(\omega, T)$$

where $P_\omega \, d\omega$ is the radiation intensity in the frequency range ω to $\omega + d\omega$, A_ω varies from unity for a blackbody to zero for a non-absorbing body. The radiation from the plasma is compared with a known variable radiating standard radiating through the plasma to a detector.

The plasma temperature is denoted by T_p and the variable by T_s. If A_ω is the absorptivity of the plasma, $1 - A_\omega$ is the transmissivity of the plasma. The power detected is (see Fig. 1).

$$P_{01} = (1-A_\omega) B(\omega T_s) + A_\omega B(\omega T_p) \tag{63}$$

Without the plasma the radiation observed is

$$P_{02} = B(\omega T_s) \tag{64}$$

The difference is

$$\Delta P_0 = P_{01} - P_{02} = A_\omega (B(\omega, T_p) - B(\omega T_s)) \tag{65}$$

if

$$\Delta P = 0, \text{ then } T_p = T_s$$

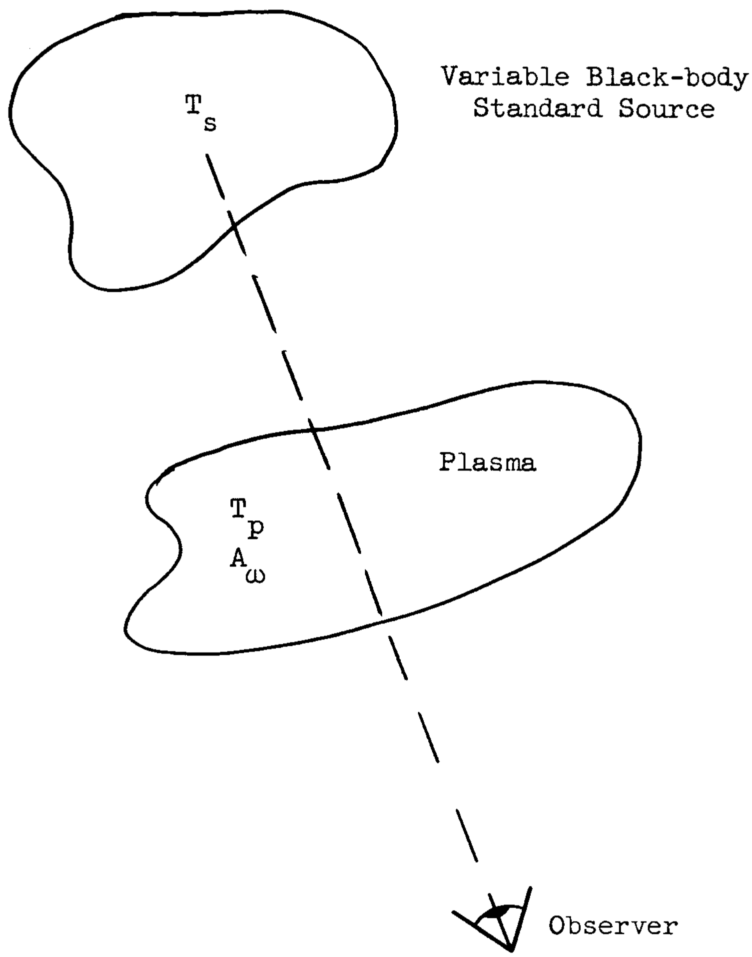


Fig. 1. Arrangement for Radiation Measurements

no matter what the value of A_ω is. By periodically inserting the plasma between the standard and the detector, the standard is adjusted until $\Delta P = 0$.[18] If the blackbody radiation term $B(\omega,T)$ is expressed as its equivalent temperature, the temperature equivalent of the microwave power at point T_1 is (see Fig. 2)

$$T_1 = T_s \frac{1}{\alpha_p} + T_o \left(1 - \frac{1}{\alpha_p}\right) \tag{66}$$

T_o is the room temperature

α_p is adjustable

T_1 is a known variable temperature

T_s is a standard noise source and equals the temperature equivalent of the microwave power of the source.

α_ω is the attenuation of the plasma

The relation between A_ω and α_ω is the following

$$A_\omega = 1 - \frac{1}{\alpha_\omega}$$

$\alpha_\omega = 1$ no attenuation

$\alpha_\omega = \infty$ perfect absorption

If the temperature equivalent of electrons in a plasma is T and their attenuation α_e, the power seen by the detector is

$$T_R = T_1 \frac{1}{\alpha_e} + T \left(1 - \frac{1}{\alpha_e}\right) \tag{67}$$

At the point where T_R decays through the known temperature T_1 [18]

$$T_R \left(1 - \frac{1}{\alpha_e}\right) = T \left(1 - \frac{1}{\alpha_e}\right) \tag{68}$$

At this point $T_R = T = T_1$. The sampling time for temperature determination is adjustable. The experiment is essentially a repetitive one

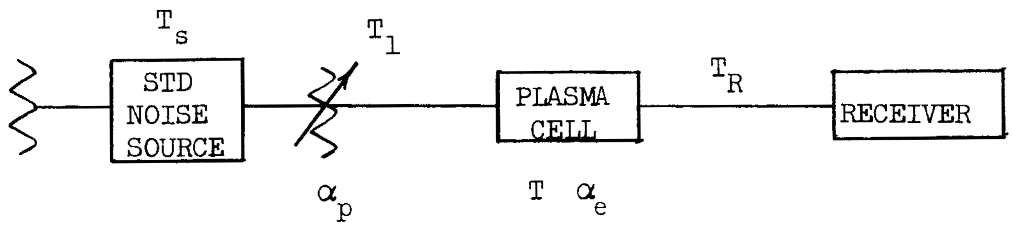


Fig. 3. Microwave Circuit for Temperature Measurements

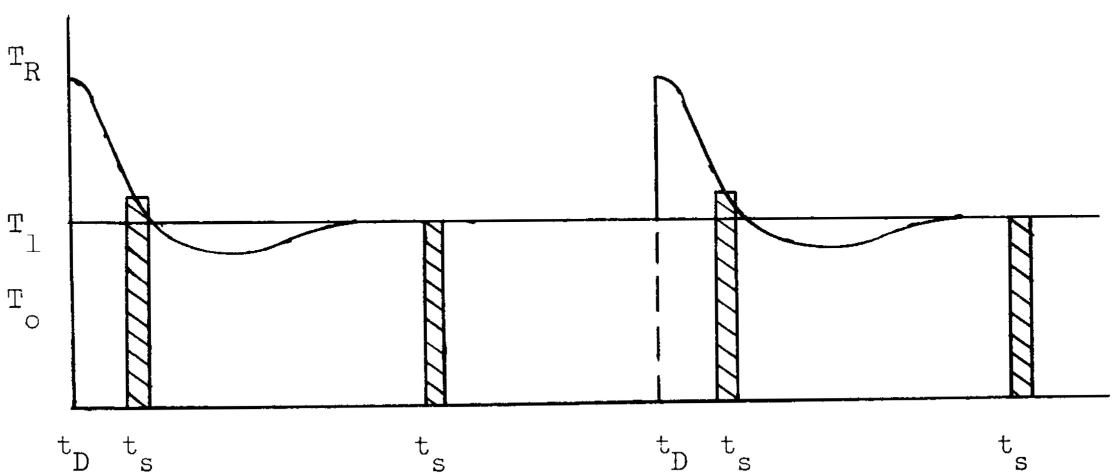


Fig. 2. Temperature Measurements Timing Diagram

at 100 cps. The discharge rate is a 100 cps. This frequency gives sufficient time for the plasma to decay between the discharges.

Figure 3 indicates the microwave circuit for temperature measurement. The circuit consists of an argon discharge tube, a precision attenuator, a plasma cell and a microwave receiver. The noise standard is a blackbody at temperature T_s. The precision attenuator is at room temperature T_o and its known attenuation is α_p.

The microwave radiation detector consists of a local oscillator, a balanced crystal detector, and a 60 mc I.F. amplifier of 20 M.C. bandwidth. The I.F. of the microwave receiver is gated on at 200 cps. at the time designated t_s in Figure 2. The detected signal is the cross-hatched pulse shown in this figure. The fundamental component of this pulse train is amplified in a high gain, narrow band 100 cps. amplifier, and the result detected in a coherent phase detector. The temperature T_1 is adjusted with the precision attenuator until $T = T_1$. Now the fundamental component of the pulse train is 200 cps. A pen recorder indicates a null. The temperature sensitivity is $\pm$ 50°K near room temperature and $\pm$ 10°K between 600° and 1000°K.

E. Microwave Bridge Measurements

The wave equation for an electric field of sinusoidal time variation passing through a homogeneous lossy medium is

$$\nabla^2 \bar{E} = j \omega \mu (\sigma + j \omega \epsilon) \bar{E}$$

μ is the permeability of medium

ϵ is the permittivity of medium

In the waveguide the dominant mode is the TE_{10} mode. Field variations are of the form $e^{-\gamma z}$, where γ is the complex propagation constant

$$\gamma = \sqrt{\left(\frac{\pi}{a}\right)^2 - \omega^2 \mu_o \epsilon_o + j\,\omega\,\mu_o\,\sigma_c} \tag{69}$$

1. Phase Shift and Attenuation

$$\gamma = \alpha + j\beta \tag{70}$$

The real part of the propagation constant α is the attenuation per unit length, and the imaginary part β is the phase shift/unit length. If l_c is the length of the plasma cell, a wave is attenuated by $e^{-\alpha l_c}$ and phase shifted by βl_c. The measured phase shift is the change from the situation of the cell with plasma to that of an empty cell.

$$\phi = \beta_o l_c - \beta l_c \tag{71}$$

where,

ϕ is the measured change in the phase

β_o is phase shift constant with no plasma in the cell

The real and imaginary parts of the conductivity are

$$\sigma = \sigma_r + j\,\sigma_i \tag{72}$$

γ_{AIR} is the propagation constant in air. It is given by the relation

$$\gamma_{AIR} = \sqrt{\left(\frac{\pi}{a}\right)^2 - \omega^2 \mu_o \epsilon_o} \tag{73}$$

There is no attenuation with air as the medium in the cell. Therefore γ_{AIR} can be written

$$\gamma_{AIR} = j\,\beta_o = j\sqrt{\omega^2 \mu_o \epsilon_o - \left(\frac{\pi}{a}\right)^2} \tag{74}$$

Solving for β_o it is found

$$\beta_o = \sqrt{\omega^2 \mu_o \epsilon_o - \left(\frac{\pi}{a}\right)^2} \tag{75}$$

Substitute relations (70), (71), (72), and (75) into equation (69) and solve for the real and the imaginary parts of the conductivity

$$\sigma_r = \frac{2\alpha}{\omega\mu_o} \left[\beta_o - \frac{\phi}{l_c}\right] \tag{76}$$

$$\sigma_i = \frac{1}{\omega\mu_o}\left[\left(\frac{\pi}{a}\right)^2 - \omega^2 \mu_o \epsilon_o - \alpha^2 + \beta_o^2 - 2\beta_o \frac{\phi}{l_c} + \frac{\phi^2}{l_c^2}\right] \tag{77}$$

2. <u>The Microwave Interferometer</u>

The microwave circuit is an interferometer. The properties of the hybrid "t" junction which mixes the two signals e_2 and e_1 (see Figure 4) are that

$$e_a = \frac{e_2 + e_1}{2} \tag{78}$$

$$e_b = \frac{e_2 - e_1}{2} \tag{79}$$

e_1 is the reference signal $= E\, e^{j\omega t}$ (80)

e_2 is the reference signal shifted and attenuated by the plasma (see Figure 5).

$$e_2 = E\, e^{j\omega t} \; e^{-A} \; e^{-j\phi} \tag{81}$$

$A = \alpha\, l_c$ is the total attenuation in the cell (82)

e_a and e_b are the output electric field intensities in the output arms of the interferometer.

Substitute relations (80) and (81) into (78) and (79) to obtain

$$e_a = \frac{1}{2} E\, e^{j\omega t} \left[e^{-A} \; e^{-j\phi} + 1\right] \tag{83}$$

$$e_b = \frac{1}{2} E\, e^{j\omega t} \left[e^{-A} \; e^{-j\phi} - 1\right] \tag{84}$$

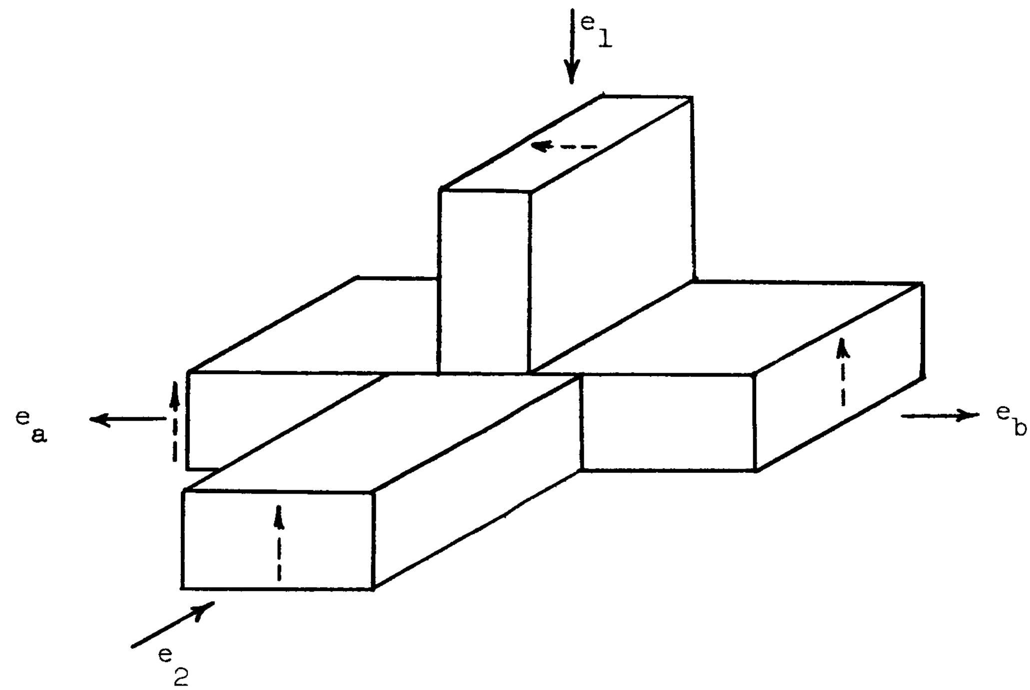


Fig. 4. Hybrid T Detector

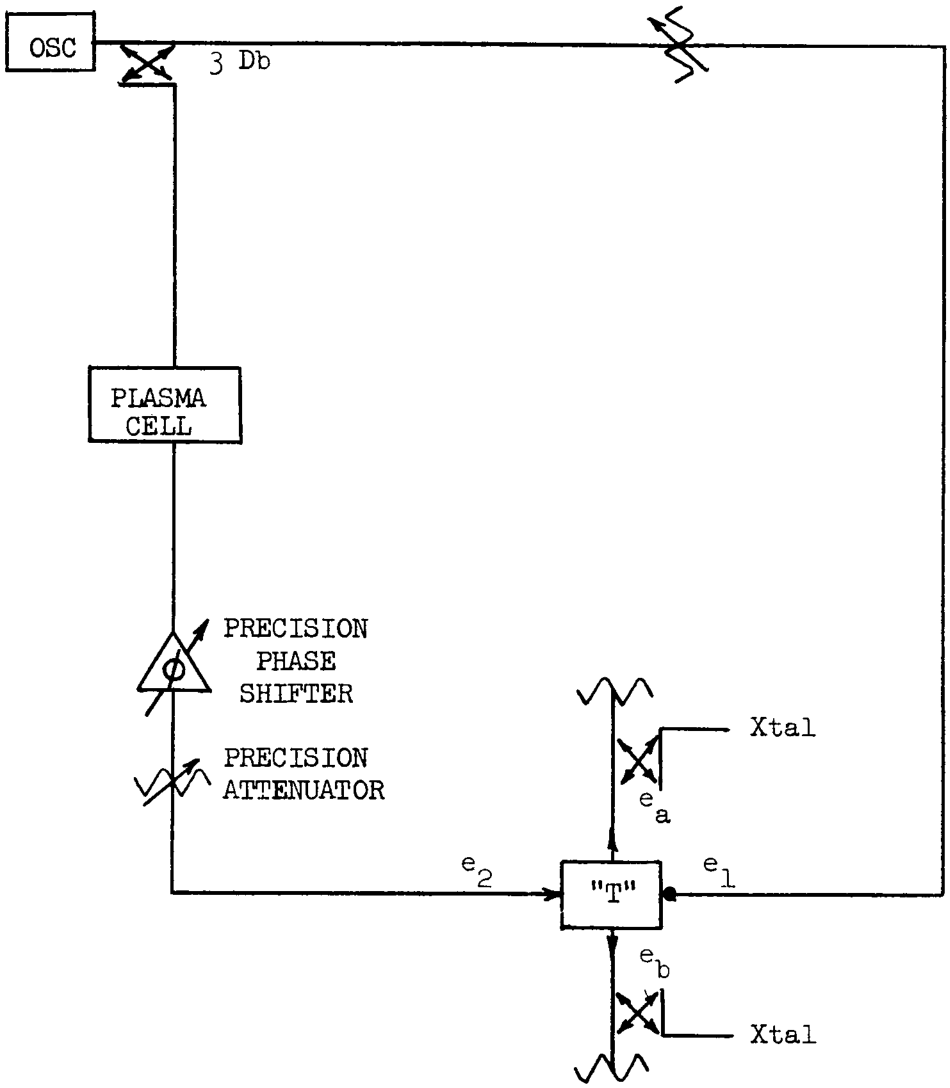


Fig. 5. Basic Microwave Bridge Circuit

The magnitudes of e_a and e_b are the quantities detected by a crystal detector. Thus

$$\left| e_a \right| = E_a \tag{85}$$

$$\left| e_b \right| = E_b \tag{86}$$

Substitute (85) and (86) into (83) and (84) and find

$$\begin{aligned} E_a &= \frac{E}{2} \left[1 + 2e^{-A} \cos \phi + e^{-2A} \right]^{1/2} \\ E_b &= \frac{E}{2} \left[1 - 2e^{-A} \cos \phi + e^{-2A} \right]^{1/2} \end{aligned} \tag{87}$$

A family of constant attenuation curves are circles (see Figure 6)

$$E_a^{\ 2} + E_b^{\ 2} = \frac{E^2}{2} (1 + e^{-2A}) \tag{88}$$

of radius

$$R = \frac{E}{2} (1 + e^{-2A})^{1/2}$$

A family of constant phase shift lines are quartics.

$$E_a^{\ 4} - 2 E_a^{\ 2} E_b^{\ 2} + B_b^{\ 2} = E^2 \cos^2\phi \, (E_a^{\ 2} + E_b^{\ 2}) - E^4 \cos^2\phi \tag{89}$$

The actual calibration is somewhat distorted owing to the nonlinearities of the crystal detector (see Figure 7).

The microwave has a negligible effect on the plasma distribution and average energy. The criterion is obtained from the energy balance equation[2]

$$\frac{d\langle u \rangle}{dt} = G \nu_m (\langle u \rangle - \langle u_g \rangle) = \frac{\sigma_r E_2}{2n} \tag{90}$$

where

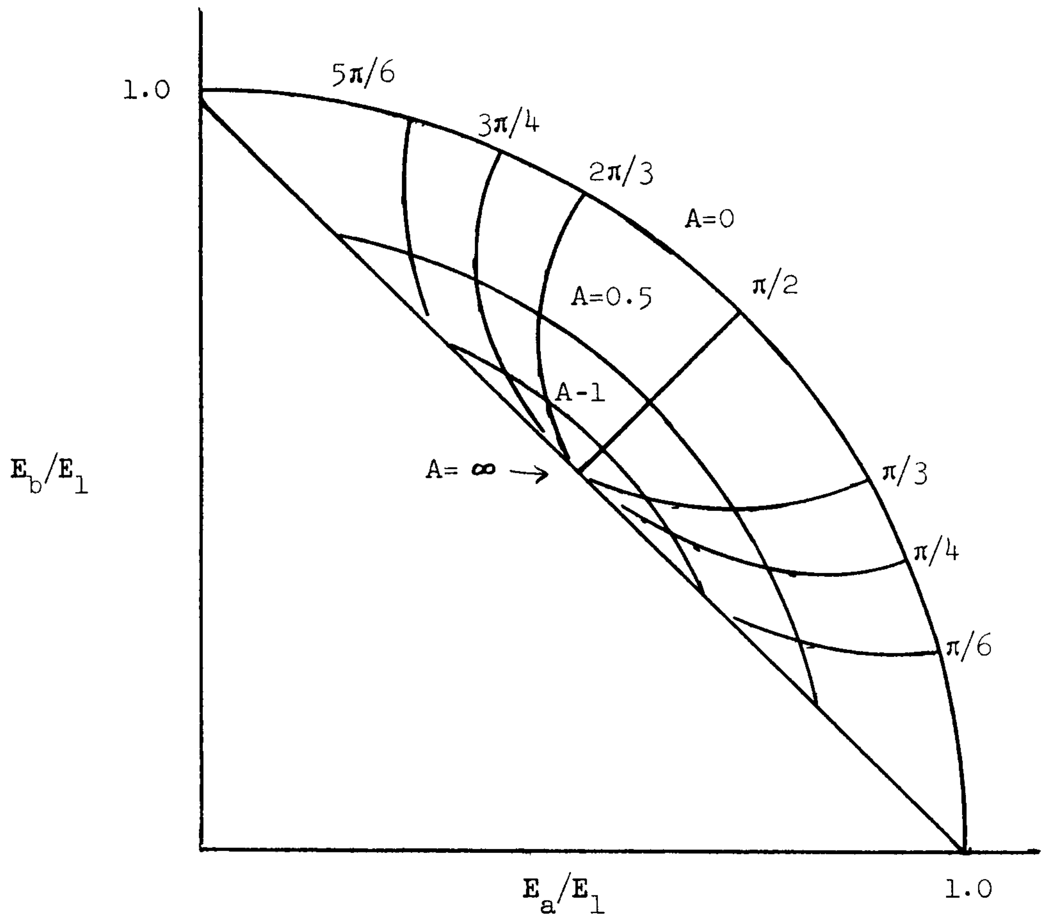
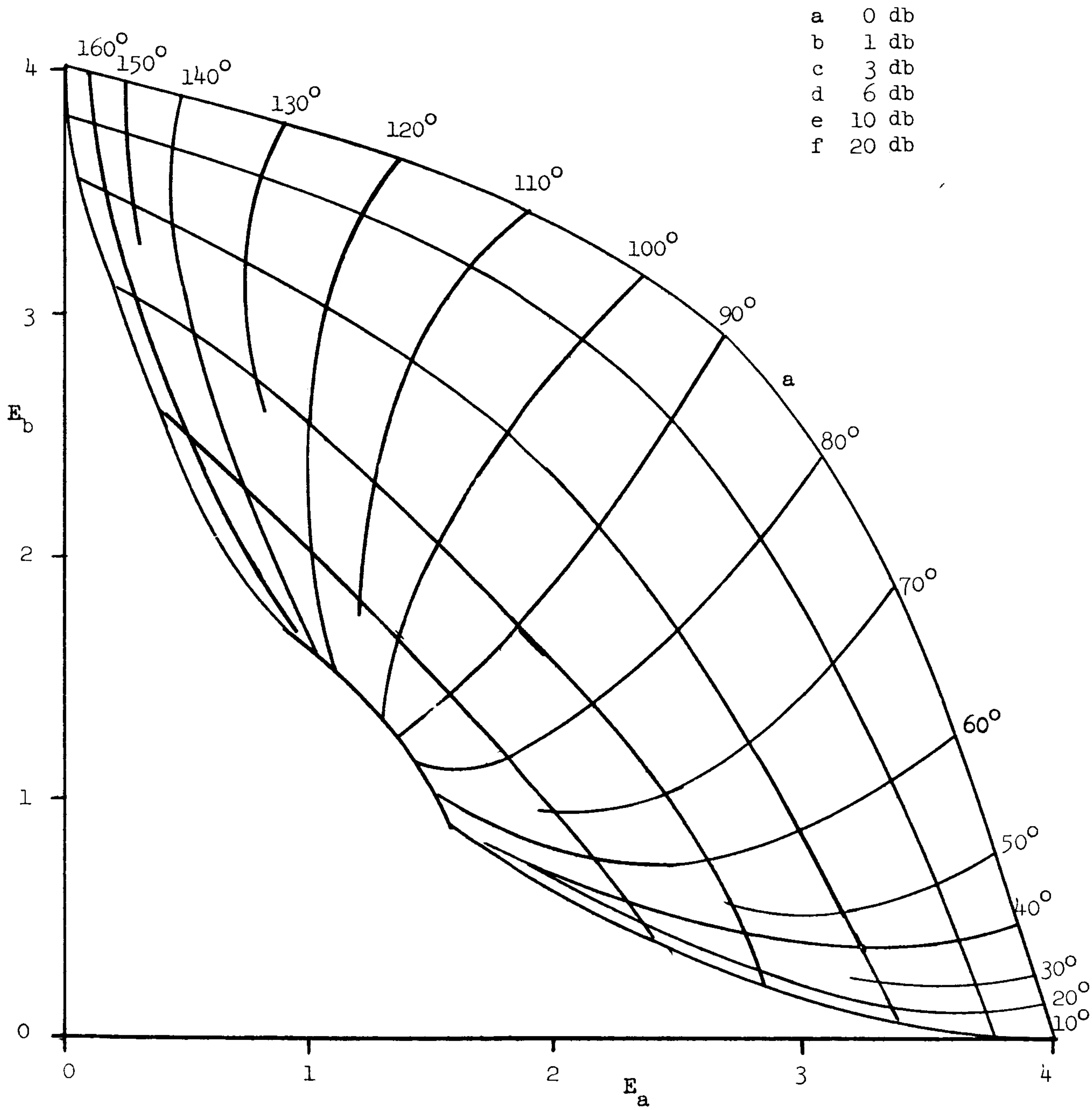


Fig. 6. Hybrid T Characteristics



Units of E_a and E_b are "Centimeters of Oscilloscope Deflection"

Fig. 7. Microwave Bridge Calibration

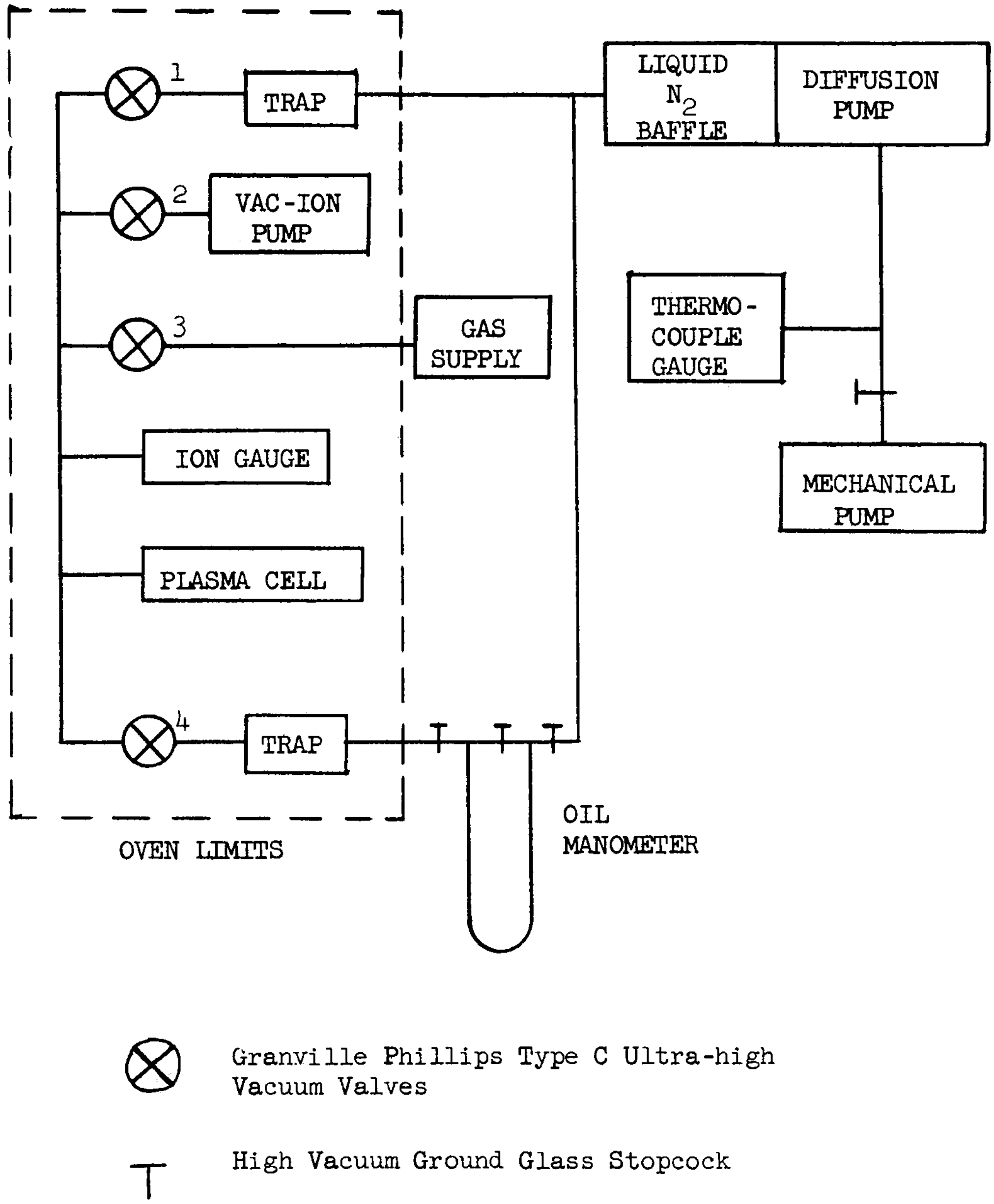


Fig. 8. Vacuum System

G is $\frac{2m}{M}$

$< u_g >$ is $\frac{3}{2}\, k\, T_g$

T_g is the gas temperature

E is the maximum microwave amplitude

The criterion is obtained from the condition

$$G \nu_m \left(< u > - < u_g > \right) >> \frac{\sigma_r E^2}{2n} \qquad (91)$$

$$E << 2.1 \times 10^{-8} \left(\omega^2 + \nu_m^2\right)^{1/2} \text{ volts/meter} \qquad (92)$$

$E <<$ 1210 volts/meter in our case

$\omega = 9.152 \times 10^9 \times 2\pi$ rad/sec

corresponding to 157 milliwatts for the TE_{10} mode. The maximum power passed through the plasma is 5% of the critical value.

3. Attenuation Measurements

In the late afterglow the attenuation is very small. The phase shift however is not negligible and can be measured with very good accuracy into the late afterglow by assuming it lies on the 0 db curve (actually very close to it). Even though the attenuation is very close to zero it must be measured to make the calculations since the real part of the conductivity depends on the product of α and β. To measure the attenuation, straight attenuation measurements are made. That is the one arm of the interferometer with the plasma is used, since we only need to measure one parameter. There is a crystal detector in this arm placed just after the plasma cell. Since the microwave signal is amplitude modulated by an audio square wave, there is a zero reference signal on the oscilloscope. The whole pattern can be conveniently displayed on the 500 mv/cm scale. When the plasma is turned on it attenuates the signal. As is seen from Figure 9 it is necessary to amplify this attenuation. However at higher gain the top part of the signal goes off the scale vertically and cannot be brought down by the vertical position control. To bring the signal down to be displayed, a small negative voltage must be added to the signal. This task can be accomplished by a variable bias box arrangement as shown in Figure 9. A small negative voltage of correct magnitude can be impressed upon the signal until it comes into view on the oscilloscope. As soon as a picture of the attenuation curve has been made a calibration must be obtained with the plasma off. By this method we are able to use the 10 mv/cm scale on the oscilloscope and detect .02 db attenuation. Also see Figure 10.

F. Relaxation Times

The mean decrease in electron energy assuming no microwave heating is

$$- \frac{d\langle u \rangle}{dt} = \frac{2m}{M} \left(\langle u \rangle - \langle u_g \rangle \right) \nu \tag{93}$$

The mean free path is approximately independent of the electron velocity

$$\langle \nu \rangle = \frac{v}{L_m} = p_o P_c v \tag{94}$$

Solving (94) for L_m, it is given by

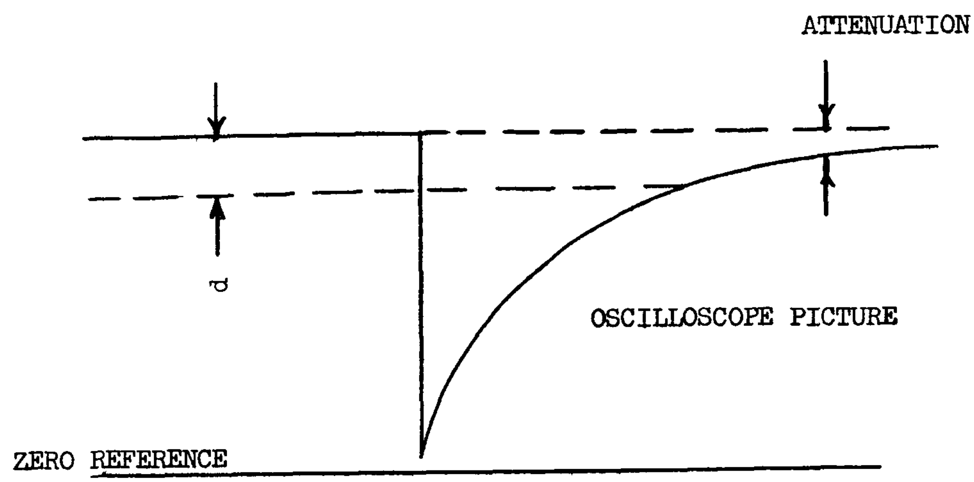
$$L_m = \frac{1}{p_o P_c} \tag{95}$$

Set

$$c = \frac{v_o - v_T}{v_o + v_T} \tag{96}$$

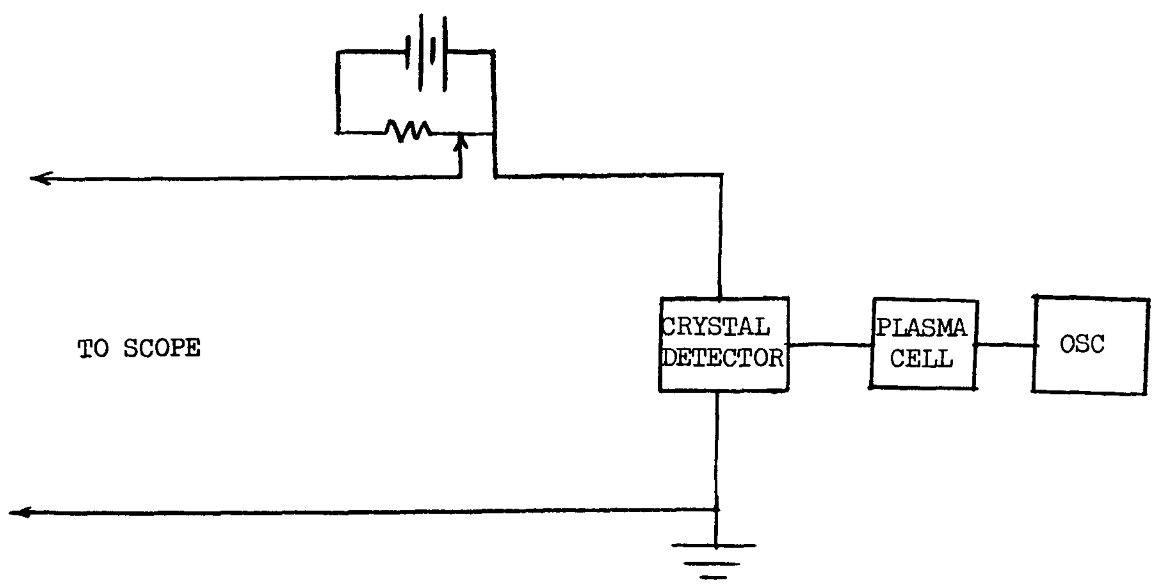
$$g = \frac{2m}{M} \frac{v_T}{L_m} \tag{97}$$

A)



A) ATTENUATION CURVE

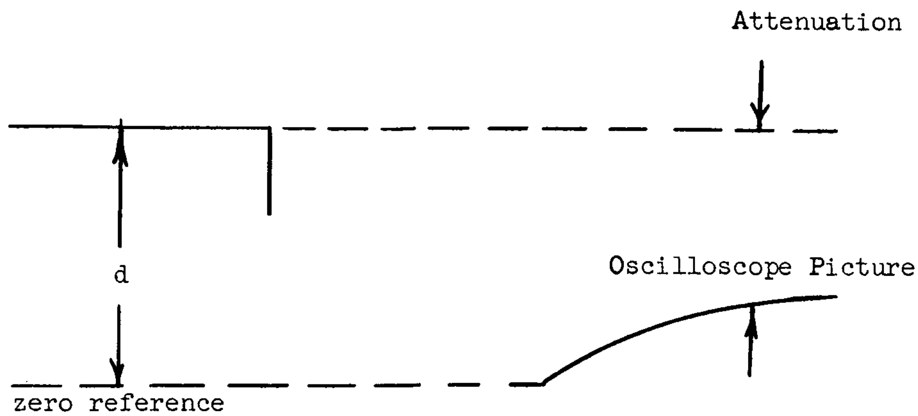
B)



B) NEGATIVE BIAS SUPPLY - NO BIAS VOLTAGE

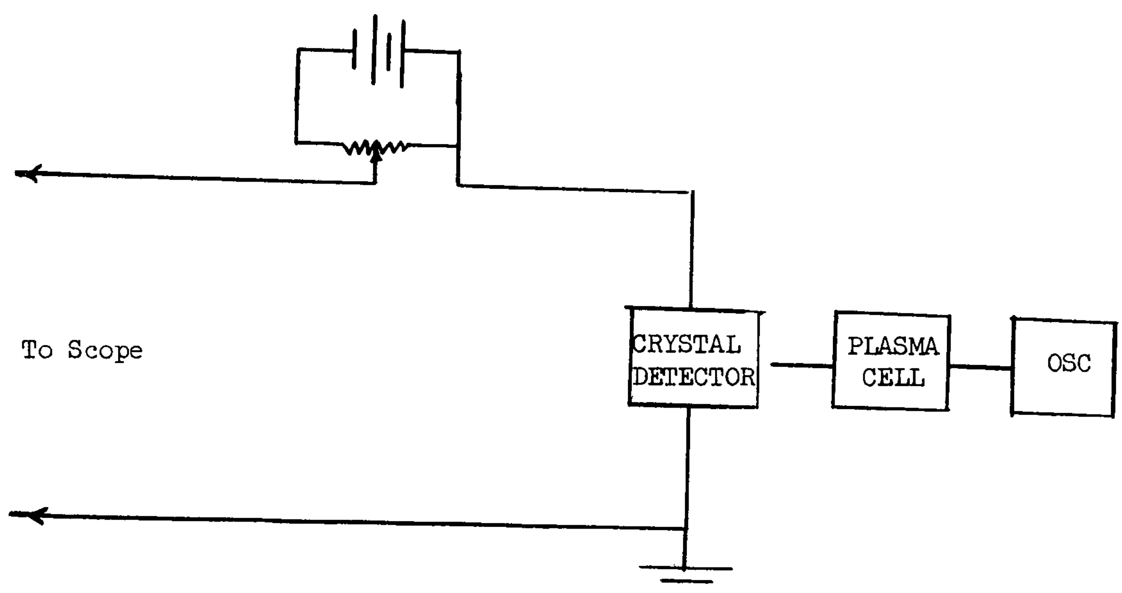
Fig. 9. Attenuation Measurement

A)



A) Attenuation Curve at High Amplification

B)



Negative Bias Supply - Negative Voltage Added to Signal

Fig. 10. Attenuation Measurement with High Amplification

v_o = electron velocity at $t = 0$

v_T = electron velocity at thermal equilibrium

then

$$v(t) = v_T \left(\frac{e^{gt} + c}{e^{gt} - c}\right) \tag{98}$$

The time t_o' for the energy $< u >$ of the electrons to deviate by less than 10% from the thermal energy of the gas is found from (98)[16]

$$t_o' = 9.2 \times 10^2 \frac{L_m A}{p' v_T} \left\{ \ln \left(\frac{v_o - v_T}{v_o + v_T}\right) + 3.7 \right\} \text{ sec} \tag{99}$$

A is the molecular weight of nitrogen = 28 in atomic weight units. If v_o and v_T are expressed in cm/sec and the first term in the brackets of (99) is neglected then

$$t_o' = 3.4 \times 10^3 \frac{L_m A}{T\, v_T} \text{ sec} = 4.5 \times 10^{-3} \frac{L_m A}{p' (T_g)^{1/2}} \tag{100}$$

T_g is the gas temperature = 300°K

L_m is obtained from the relation (95).

$$L_m = \frac{1}{p_o P_c} = \frac{1}{p' \frac{273}{T_g}} P_c \tag{101}$$

where

$$p_o = p' \frac{273}{T_g}$$

and

p' is the pressure of the gas in mm. of Hg.

Substitute equation (101) into equation (100), t_o' is then given by

$$t_o' = \frac{4.5 \times 10^{-3} A}{p' (\frac{273}{T_g}) P_c \; p' (T_g)^{1/2}} \tag{102}$$

or

$$t_o' = \frac{4.5 \times 10^{-3} A T_g^{1/2}}{p'^2 P_c \;\; 273} = \frac{8.0}{p'^2 P_c} \times 10^{-3}$$

P_c is about 15 (cm Torr)$^{-1}$ for nitrogen.

So that

$$t_o' = \frac{5.3 \times 10^{-4}}{p^2} \tag{103}$$

if

Table 1

then

$p' = 5.6$, mm	4.2, mm	2.7, mm	1.75 mm
$t_o' = 17\ \mu$ sec	30 μ sec	72 μ sec	170 μ sec

A time constant for the relaxation to a Maxwellian distribution can be calculated, since it is the collisions between the molecules of a gas which cause the distribution function to be Maxwellian. The distribution function of a gas whose molecules are simply left to collide among themselves is Maxwellian.

If the gas is disturbed slightly so that f_1 has some value f_o' then f_1 will become zero when a collision occurs.[12]

Thus $f_1 = f_o' \times$ (probability a molecule will travel a time t without suffering a collision)

$$f_1 = f_o' \times p(t) \tag{104}$$

where

$$p(t) = e^{-\nu_m t}$$

Therefore

$$f_1 = f_o e^{-\nu_m t}$$

The assumption that preferred motion is destroyed indicates the use of collision frequency for momentum transfer. We therefore introduce a relaxation time between such collisions. Thus

$$\tau_m = \frac{1}{\nu_m} \tag{105}$$

The collision frequency is of the order of

$$\nu_m \sim 10^8/\text{sec} \quad \text{to} \quad 10^9/\text{sec}$$

$$\tau_m \sim .01\ \mu\ \text{sec} \quad \text{to} \quad .001\ \mu\ \text{sec}$$

Immediately following the discharge a Maxwellian distribution should be formed in the absence of source electrons due to metastable heating.

Within the decaying plasma of an afterglow, there are effects which tend to destroy the Maxwellian distribution of electrons. Electron energy approaches room temperature at a rate determined by effects which tend both to increase and decrease the average electron energy.[7] Electron energy is removed by the elastic recoil collisions between electrons of mass m and molecules of mass M, from which the electron loses a part $G = 2m/M$ of its energy per collision.[1] Electron-ion recombination preferentially removes low energy electrons and increases average electron-free energy. Metastable-metastable interactions are a source of high energy electrons. Metastable-metastable interactions tend to destroy the Maxwellian distribution. A process that contributes to cooling of the electrons is diffusion cooling.

PART III

EXPERIMENTAL RESULTS

A. Introduction

The real and imaginary parts of the conductivity were obtained by use of equation (53) and (54). The data was compiled on an IBM 1410 computer. The collision frequency was then obtained from the equation (42)

$$\frac{\sigma_r}{\sigma_i} = -\frac{4}{3}\frac{\langle \nu \rangle}{\omega} \qquad (42)$$

where $\omega = 9.153 \times 10^9$ cycles/sec x 2π rad/cycle.

P_c was calculated on the basis of a hard sphere model interaction between electrons and nitrogen molecules, and was given by averaging equation (34) for constant P_c

$$P_c = \frac{\langle \nu \rangle}{\langle v \rangle p_o}$$

or

$$P_c = \frac{1.82 \times 10^{-8} \langle \nu \rangle}{\langle u \rangle^{1/2} p_o} \text{ (cm.-Torr)}^{-1} \qquad (106)$$

$\langle u \rangle$ is related to the electron radiation temperature by the relation

$$\langle u \rangle = \frac{3}{2} kT = \frac{1}{2} m v_{rms}^2$$

v_{rms} is the root mean square speed of the electrons and is given by

$$1.08 \langle v \rangle = v_{rms}$$

The temperature in the laboratory was 20°C. $\langle u \rangle$ was calculated over the range .040 - .078 ev. The values obtained for P_c agree remarkably well to those obtained by Phelps et al[17] rather than those of Anderson and Goldstein[3].

It is also revealed that metastable collisions have a marked effect in determining the velocity distribution in the early afterglow.

B. Electron Temperature Measurements

It is seen from the temperature vs time measurements that the time to relax within 10% of the room temperature is much longer than predicted by theory (see Table 1). It is approximately two orders of magnitude longer. The higher the pressure, the faster the relaxation to equilibrium which is an expected result (see equation (103)). A metastable effect therefore heats the plasma during the afterglow.

Another interesting effect is the slight increase in temperature in the early afterglow (10 - 20 microseconds). See Figures 18 and 19. Thus the electron source is raising the temperature by injecting highly energetic electrons into the afterglow faster than energy is lost by collisions. It is to be expected that the metastable source of high energy electrons is of strongest intensity during the early afterglow, and decays with a characteristic time constant.[7] Thus the distribution function should be maintained strongly non-Maxwellian during the early afterglow.

As room temperature is approached the electron temperature measurements could only be made within $\pm$ 50°K. This fact is illustrated on the temperature graphs.

C. Probability of Collision

P_c was calculated over the energy .040 - .078 ev. P_c should experimentally calculate out as a constant throughout this range of energies, if the electron distribution function does not change with time. This result should be expected since slow electrons are dealt with. However, P_c only approaches a constant as the temperature decays to room temperature at a rate depending on pressure. The final constant value is approached much more rapidly at higher pressures. The characteristic time to attain a Maxwellian distribution of electrons is therefore the time it takes P_c to adjust to a constant. It can therefore be surmised that collisions tend to destroy the metastable electron source much more readily at higher gas densities. For example at 5.6 mm (see Figure 12) the relaxation to a Maxwellian distribution is less than 45 microseconds. Approximately 100 μ seconds elapses before P_c becomes a constant at 4.2 mm (see Figure 12). At 2.7 mm it takes a 140 microsec before P_c reaches a constant (see Figure 13). The plotted points P_c obtained at 2.7 mm and 1.75 mm must be received with caution since the perturbing effect of metastables is still large. Also at these lower pressures, diffusion destroys the uniform spatial distribution of electrons across the waveguide. So far in this discussion electron density has not been mentioned. Mainly due to the fact it cancels out of the computation. The electron density is of the order of 10^{10} electrons/cm^3 with this experimental set up.[18] The neutral density is about 3×10^{16} neutrals/cm at 1 mm pressure. The neutral density is a factor of 10^6 greater than that of the electrons and ions. However the effects of electron-ion collisions may not be negligible at the lower pressures and the theory would have to be modified accordingly.

The average value of probability of collision over energy range is obtained by averaging the experimentally measured values at 4.2 Torr and 5.6 Torr. The value of 18.1 ± 1.6 (cm Torr)$^{-1}$ agrees within 15% of that calculated by Phelps at these energies (see Figure 14).

An attempt was made to find a rigorous expression for $P_c(v)$ over the range .045 to .065 ev using equation (35). Three points were chosen from Figure 15 from the 4.2 mm curve. It must be remembered that $\langle u \rangle = \frac{3}{2}$ w.

$-\frac{1}{p_o}\frac{\sigma_r}{\sigma_i}$	$\langle u \rangle$ energy ev	Time microsec following discharge
.03079	.0655	35
.008074	.0533	70
.004190	.04183	140

Using

$$-\frac{1}{p_o}\frac{\sigma_r}{\sigma_i}(\langle u \rangle) = a_1\langle u\rangle^{1/2} + a_2\langle u\rangle^{3/2} + a_3\langle u\rangle^{5/2}$$

at the three points in the table the coefficients obtained were

$$a_1 = .614$$

$$a_2 = -39.4$$

$$a_3 = 660$$

$$P_c = 1292\left[\frac{a_1}{2} + \frac{a_2}{6}u + \frac{a_3}{24}u^2\right] \qquad (107)$$

P_c can be calculated from (107) but the numbers vary over too great a magnitude. Thus this is another indication that the distribution function must be different from Maxwellian at this pressure during the early afterglow.

D. Summary

The feasibility of measuring the probability of collision of slow electrons over a small energy range by means of a microwave interferometer and microwave radiometer has been demonstrated. The effect of metastable-metastable interactions is important during the early afterglow. The time taken for the plasma electrons to relax to a Maxwellian distribution varies inversely as the gas pressure. Much more research has to be done before the actual metastable-metastable processes are fully understood. One means for extending the energy range over which the probability of collision is measured would be to take the data at various gas temperatures. This task can be accomplished by placing the plasma cell in an oven during the experiment. The value of P_c measured in this experiment was $P_c = 18.1 \pm 1.6$ (cm Torr)$^{-1}$ over the energy range .040 to .078 ev.

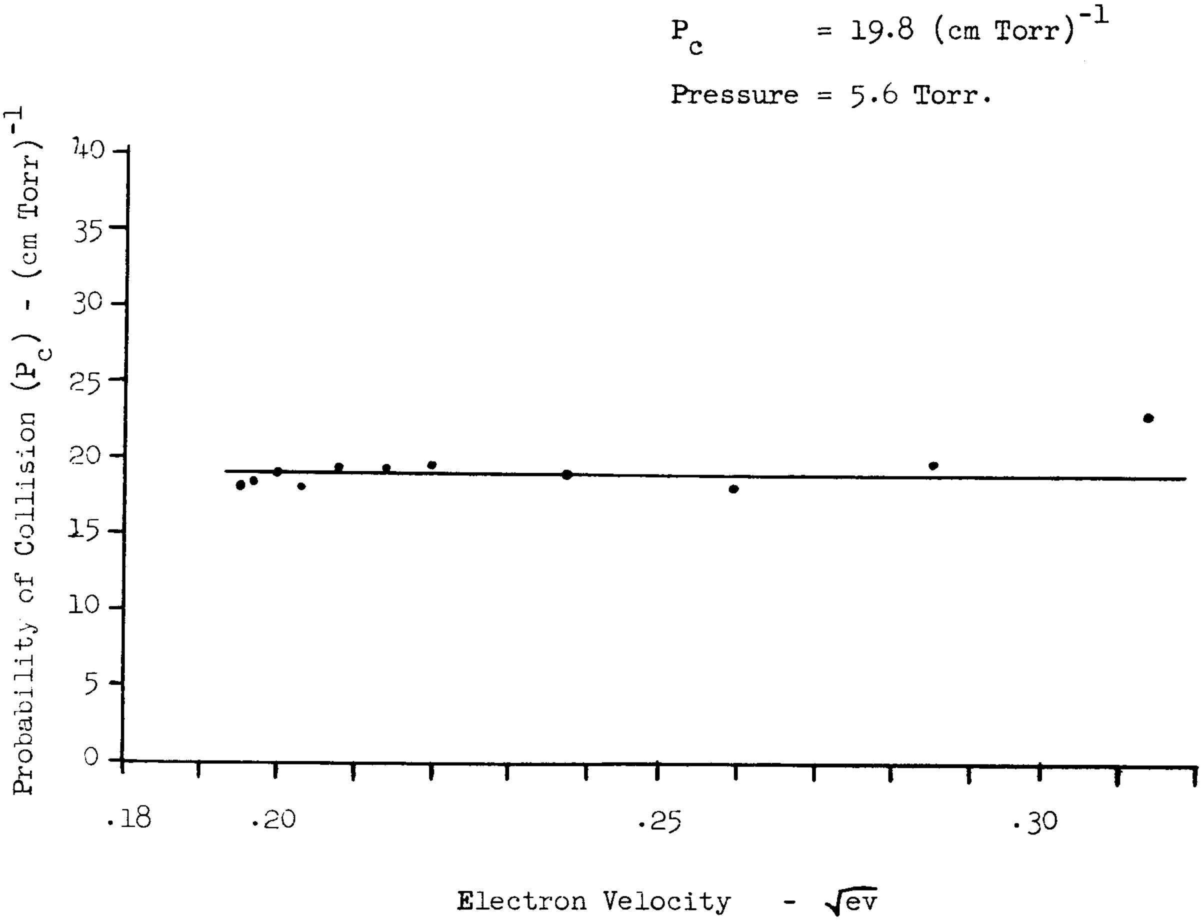


Fig. 11. Probability of Collision as a Function of Electron Velocity at 5.60 Torr.

P_c = 16.6 (cm Torr)$^{-1}$

Pressure = 4.2 Torr.

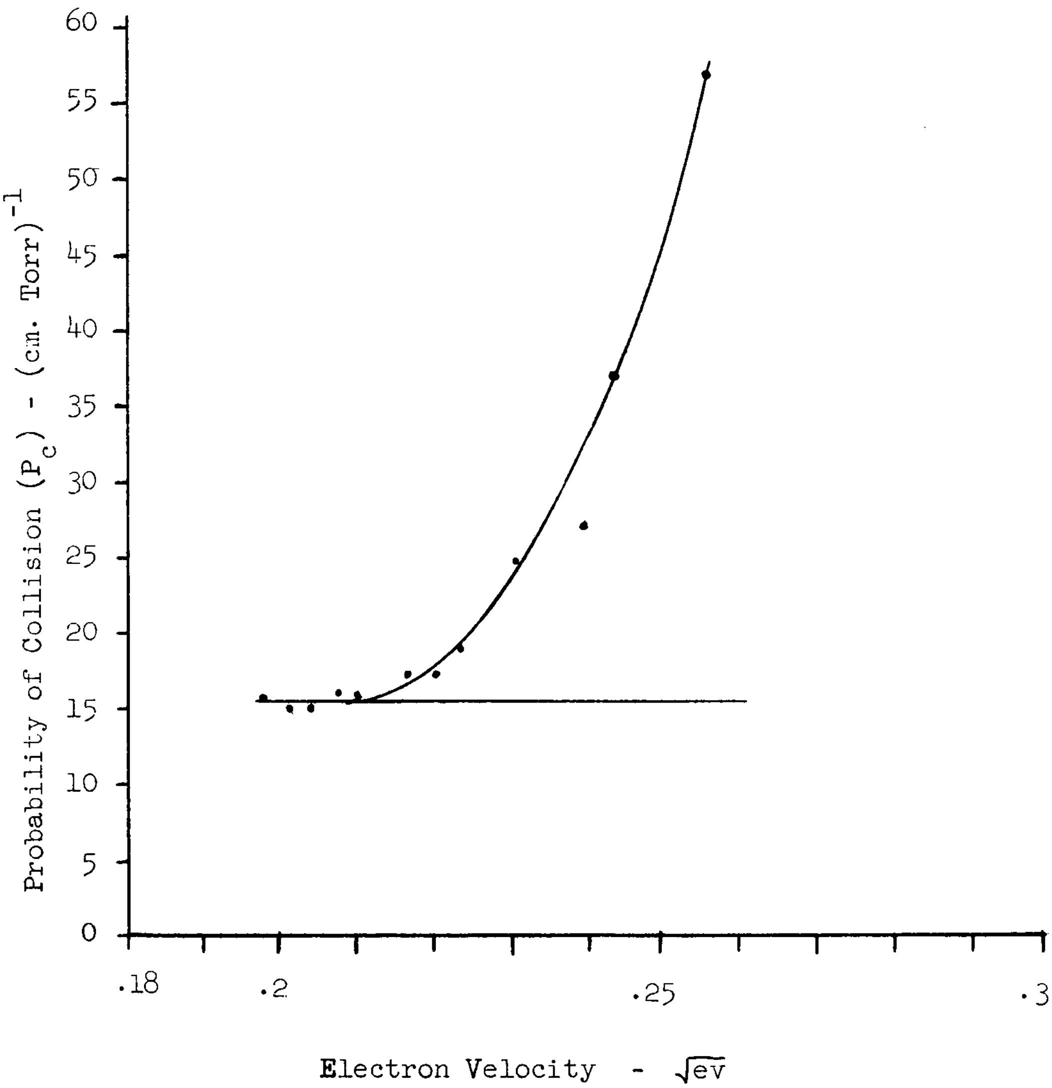
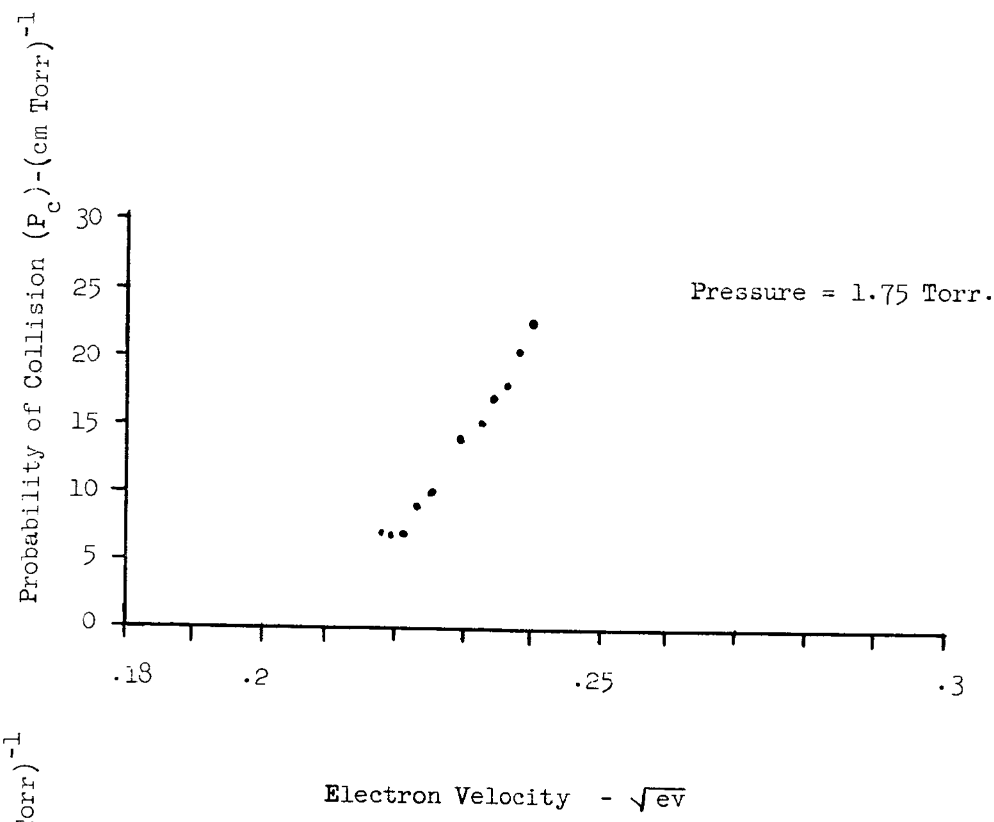


Fig. 12. Probability of Collision as a Function of Electron Velocity at 4.2 Torr.



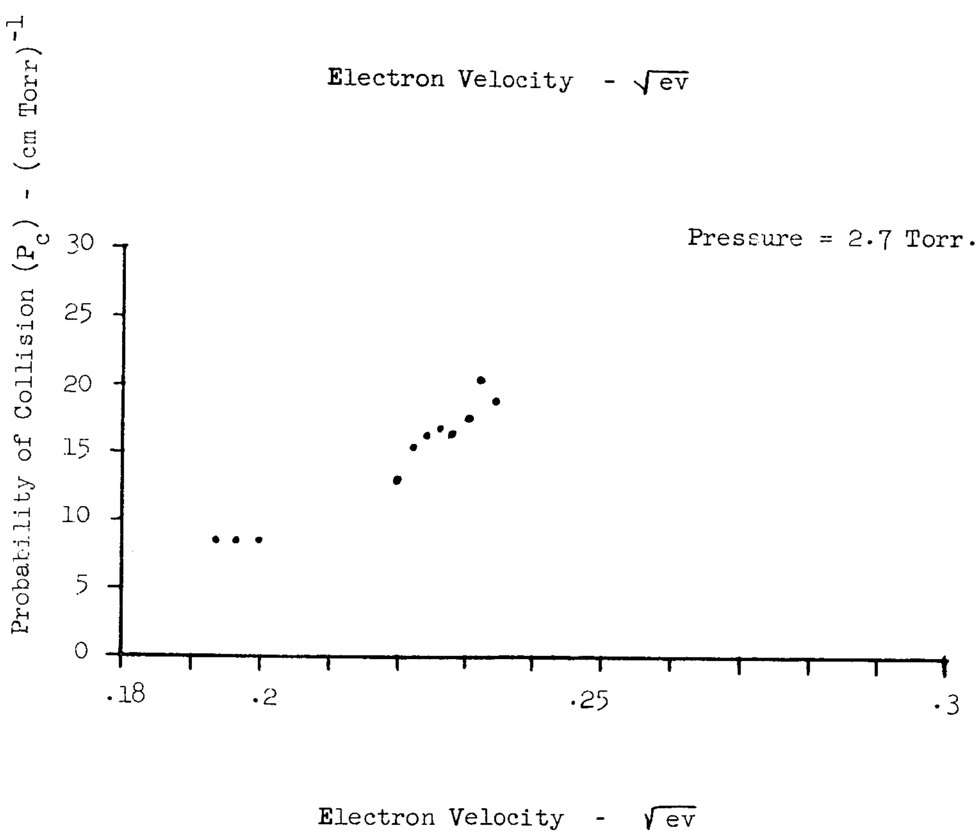


Fig. 13. Probability of Collision as a Function of Electron Velocity at 2.70 Torr and 1.75 Torr

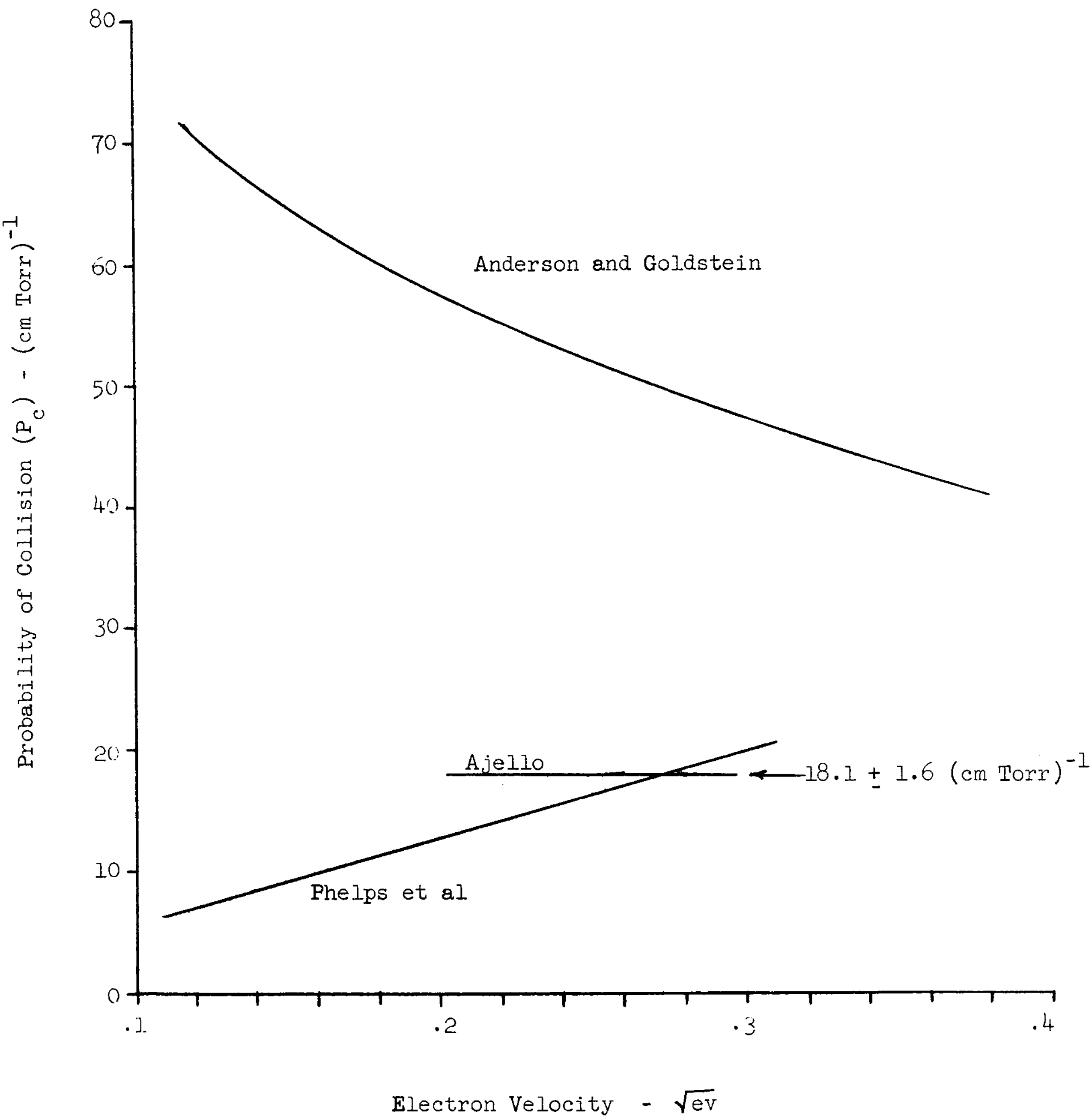


Fig. 14. Probability of Collision as a Function of Electron Velocity According to Various Authors

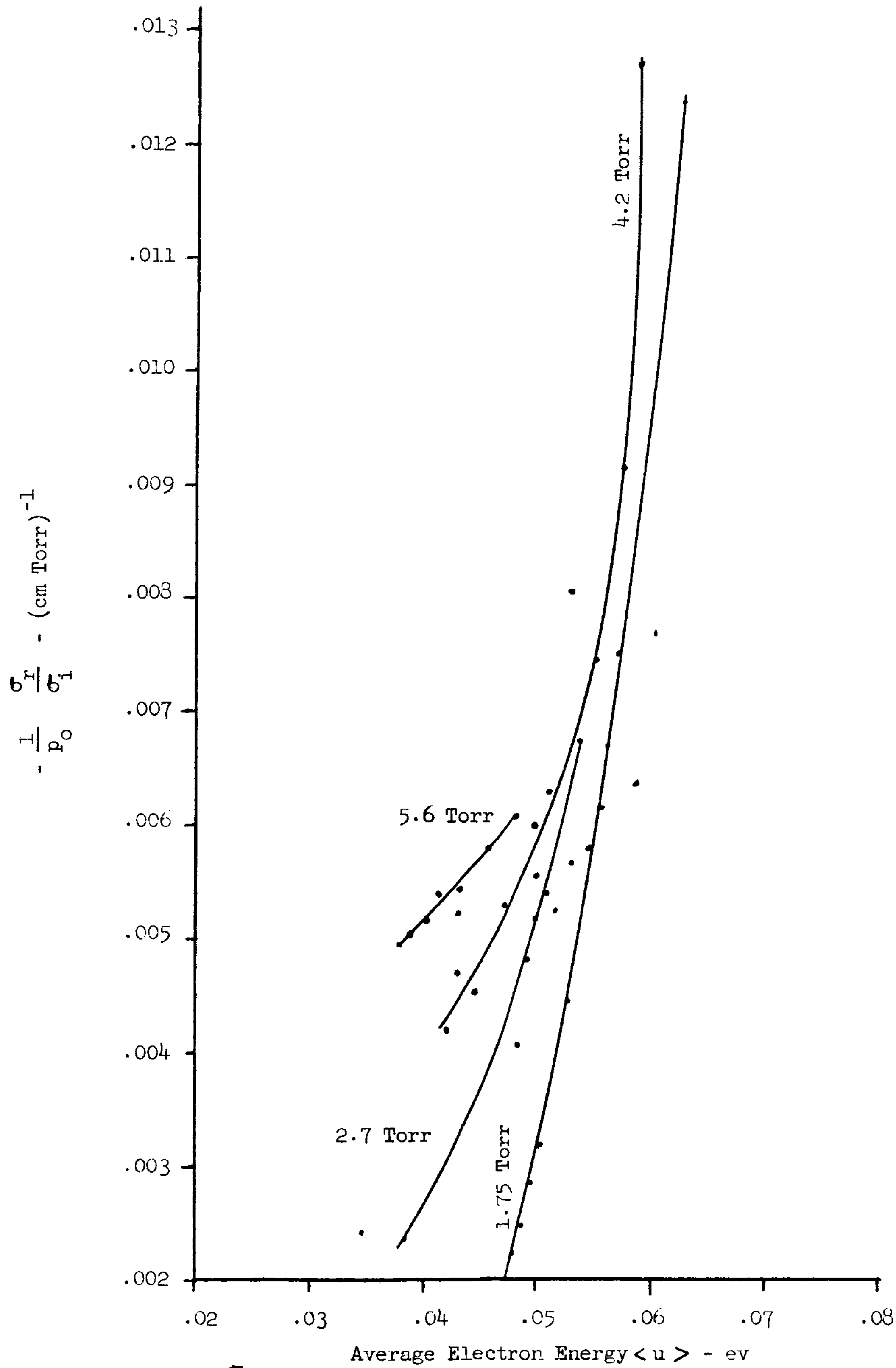


Fig. 15. $-\frac{1}{p_o}\frac{\sigma_r}{\sigma_i}$ as a Function of Electron Energy at Various Pressures

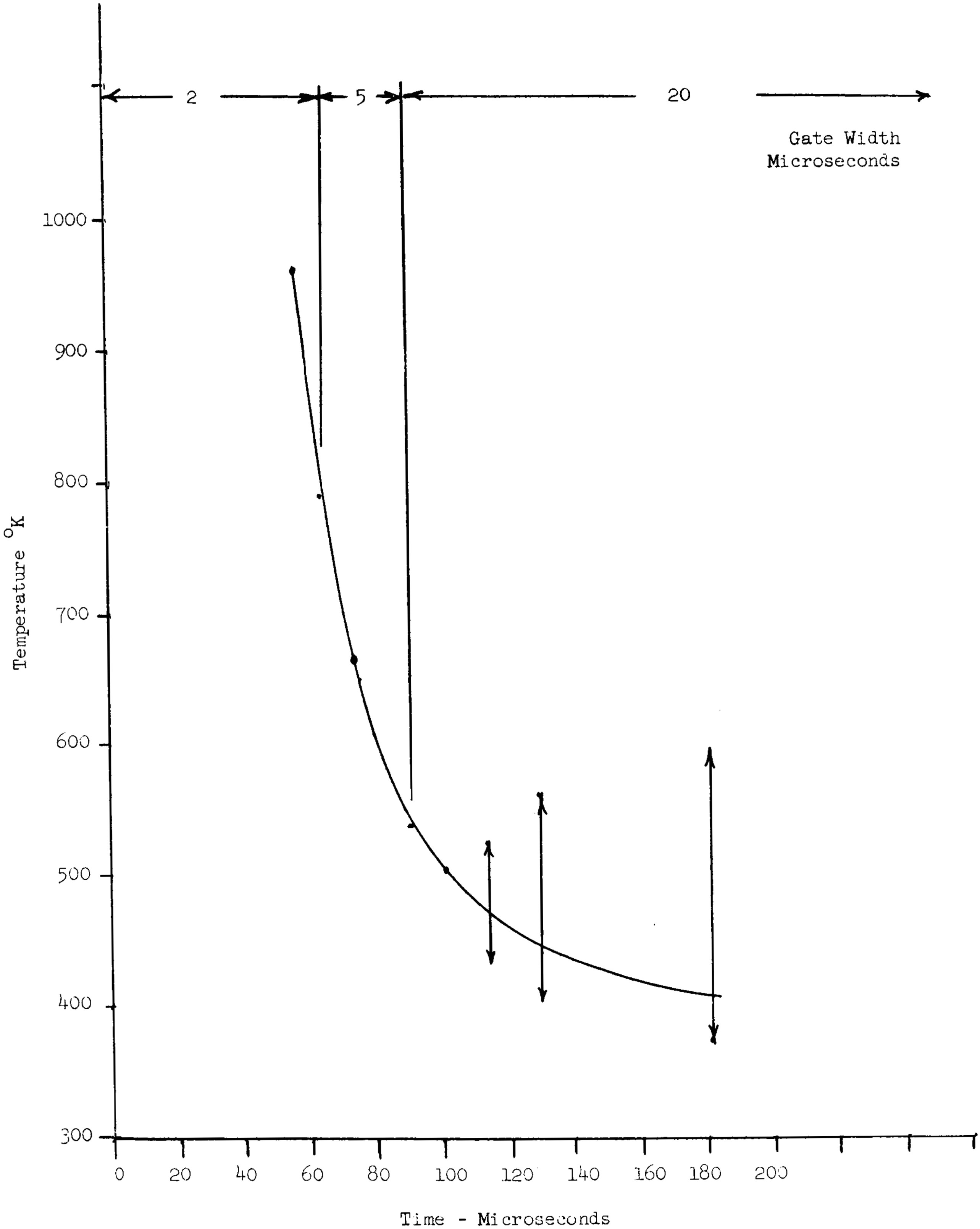


Fig. 16. Decay of Electron Gas Temperature at 5.6 Torr

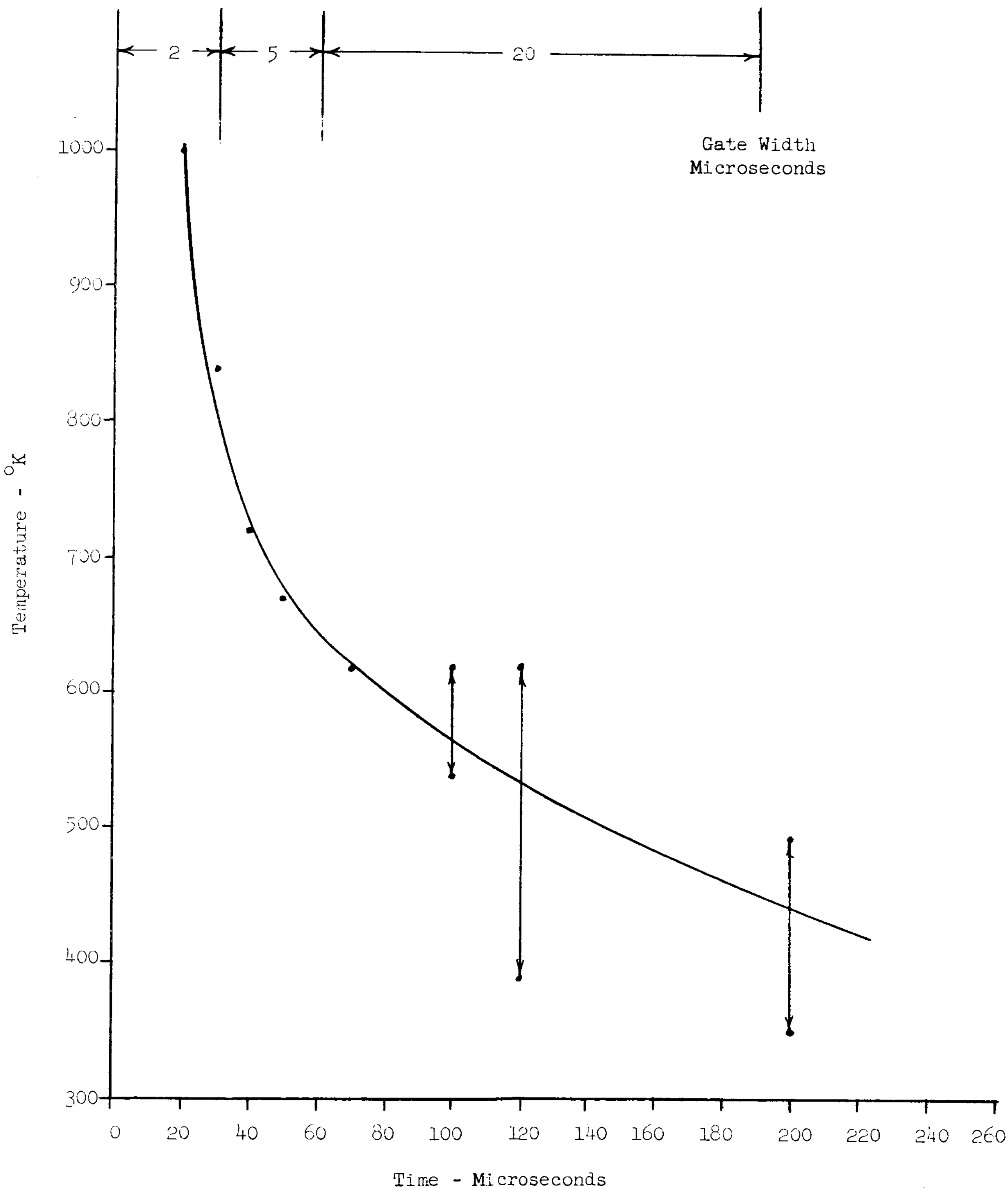


Fig. 17. Decay of Electron Gas Temperature at 4.20 Torr

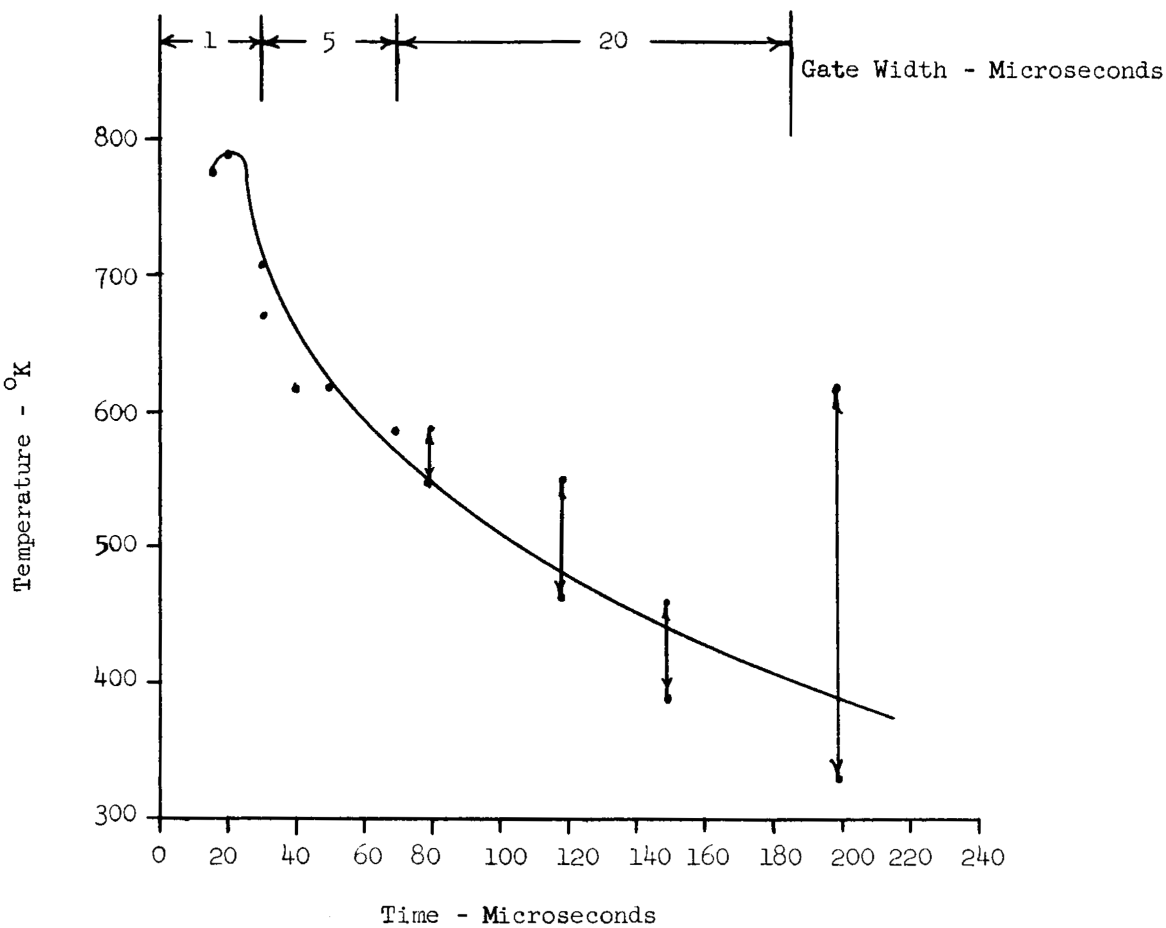


Fig. 18. Temperature Decay of Electron Gas at 2.70 Torr

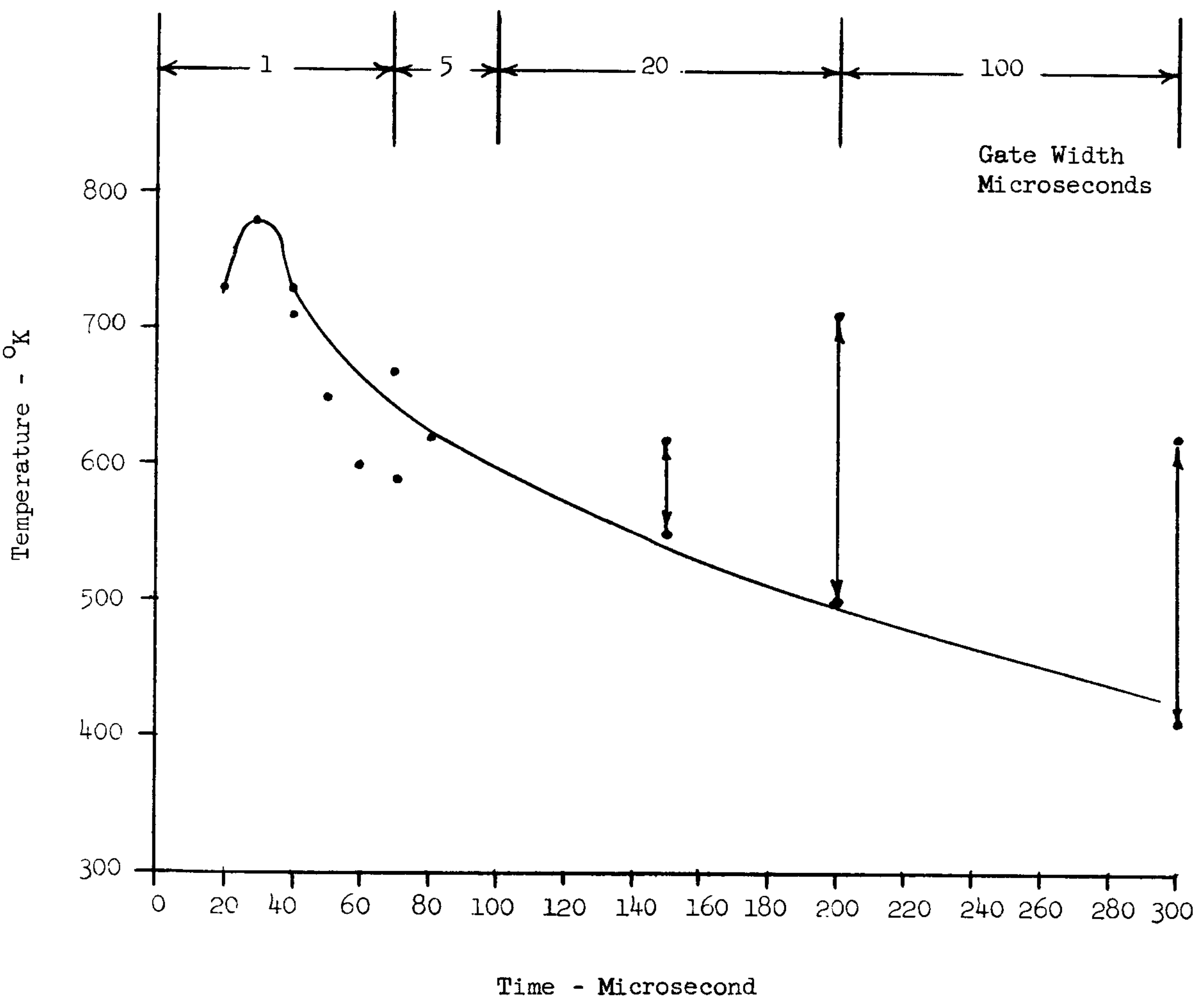


Fig. 19. Decay of Electron Gas Temperature at 1.75 Torr

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